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Robustness of latin square design against loss of pairs of treatments

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Abstract. This paper deals with the problem of finding the robustness of latin square design when pair of treatment are lost either in a row or in any two rows. It is observed that a Latin square design is fairy robust against the unavailability of one pair of treatment in any one row and also against the loss of two pair of treatments in any two rows. Further we have also obtained the lower and upper bound of the Latin square design against the unavailability of two pairs of treatment in any two rows.

Keywords. Robustness, A-Efficiency, Latin square design.

1 Introduction

Ghosh (1982) discussed method for finding robustness of design against the unavailability of data. Further, criteria of robustness has been thoroughly investigated by different authors, *viz.* Dey and Dhall (1988), Whittinghill (1989), Mukerjee and Kageyama (1990), Das and Kageyama (1992), Mukerjee and Kageyama (1990), Ghosh and Gosai (1998) *etc.*

Most of the robustness criteria against the unavailability of data are: (i) to get the connectedness of the residual design (ii) to have the variance balance of the residual design and (iii) to consider the A-efficiency of residual design for the robustness study.

So far, robustness of incomplete block designs and complete block designs are carried out against loss of either s observations in one block. However sometimes it may happen that data are lost or not available for either a pair of treatments in any one row, or any two pairs of treatments in any two rows in Latin square design, which require further analysis of such designs. Hence we have carried out the robustness criteria for Latin square design against the unavailability of (i) a pair of treatments in any one row, and (ii) two pairs of treatments in any two rows.

In the present investigation, let consider a standard Latin square design d of size s. Here, we have considered Latin square designs of size five to twelve as per Cochran and Cox(1957). Let d^* be the residual design obtained when (i) a pair of treatments is lost from any one row, and (ii) two pairs of treatments are lost from any two rows of a Latin square design. Assume d^* to be connected. In this case, the criterion of robustness against the unavailability of (i) a pair of treatments in any one row, and (ii) two pairs of treatments in any two rows is the overall A-efficiency, of the residual design d^*, given by

$$e(s) = \frac{\text{Sum of reciprocal } s \text{ of non-zero eigenvalue } s \text{ of } C}{\text{Sum of reciprocal s of non-zero eigenvalue } s \text{ of } C^*} = \frac{\phi_2(s)}{\phi_1(s)},$$

where, C denotes the C-matrix of Latin square design and C^* denotes the C-matrix of residual design.

In section 2.2 robustness of Latin square design against loss of a pair of treatments in any one row is discussed. While in section 3.1, we discuss the robustness of Latin Square Design against loss of two pair of treatments in any two rows. Moreover, we carried out the lower and upper bound of over all efficiency of residual design in section 4.

2 Robustness of Latin square design against loss of a pair of treatments in any one row

In this investigation, first we obtain C matrix and non zero eigen value of C matrix of standard Latin square design. Next residual design will be obtained by deleting a pair of treatments from any one row of a Latin square design. Further, C^* matrix and non zero eigen value of C^* matrix of residual design are obtained. Next we will show that Latin square designs are fairly robust against the unavailability of a pair of treatments which is discussed in Theorem 2.1.

2.1 C-matrix and nonzero eigenvalues of C-matrix of Latin square design

Consider a standard Latin Square Design d of size s. It follows that the C-matrix of d can be written as

$$C = \begin{bmatrix} s-1 & -1 & \cdots & -1 \\ -1 & s-1 & \cdots & -1 \\ \vdots & & \ddots & \vdots \\ -1 & -1 & \cdots & s-1 \end{bmatrix}_{s\times s} = s[I_s - (\frac{1}{s})E_{ss}]$$

The nonzero eigenvalues of C matrix are s with multiplicity $(s-1)$. (2.1)

2.2 Residual design obtained from Latin square design

Consider a standard Latin square design d of size s. Let a pair of treatment is lost from any one row of this Latin square design. Call this design as a residual design. The residual design becomes an incomplete block design with two types of rows and two types of columns. The size of the row, where a pair of treatments is lost, is $(s-2)$, and the size of the remaining rows retain.

Next, here, row is considered as block. The resulting incomplete block design has the parameters, $v = s$, $b = s$, $r_1 = s$ (for those treatments which are not lost), $r_2 = s-1$ (for lost treatments), $k_1 = s$, $k_2 = s-2$. Let the residual design be a connected design. For the residual design, pair of treatments, say, λ_1, λ_2 occurs together in following two ways

1. $\lambda_1 = s$, for those treatments, which are not lost and

2. $\lambda_2 = s-1$, either for those treatments, which are lost in the affected row or for (i, j) treatments, $(i \neq j)$, where i represents a treatment available among the lost treatments and j represents that treatment, which is not lost in the affected rows.

2.3 C^*-matrix and nonzero eigenvalues of C^*-matrix of residual design

Using $C^* = R^* - N^*K^{-1^*}N^{*\prime}$ we can obtain the C^*-matrix of the residual design as

$$k(k-2)C^* = \begin{bmatrix} A & D \\ D' & B \end{bmatrix}_{s\times s} \tag{2.2}$$

where A is a 2×2 sub-matrix and expressed as

$$A = s(s-1)(s-2)I_2 - (s-1)(s-2)E_{22}$$

B is a $(s-2)\times(s-2)$ square symmetric sub-matrix and expressed as

$$B = s^2(s-2)I_{(s-2)} - [s+(s-1)(s-2)]E_{(s-2)\times(s-2)}$$

D is a $2\times(s-2)$ sub-matrix and expressed as

$$D = -(s-2)^2 E_{(s-2)\times(s-2)}.$$

The non zero eigenvalues of C^*-matrix of residual design are

$$(s-1) \text{ and } s \text{ with multiplicity 2 and } (s-3) \text{ respectively.} \tag{2.3}$$

Theorem 2.1. *Latin square design of size s is fairly robust against the loss of one pair of treatments in any one row provided overall A-efficiency is given by*

$$e(s) = 1 - \frac{2}{[(s-1)^2+2]}$$

Proof. Consider a standard Latin square design of size, s whose C-matrix and eigenvalues are shown in (2.1). Now, a pair of treatments is lost from any one row of this Latin Square Design and hence design becomes residual design whose C matrix and non zero eigen values are shown in (2.2) and (2.3).

The overall A-efficiency of the residual design is obtained from

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{2.4}$$

where, $\phi_2(s)$ is sum of the reciprocal of non zero eigenvalues of C matrix of latin square design and is obtained as $\phi_2(s) = \frac{s-1}{s}$, and $\phi_1(s)$ is sum of the reciprocal of non zero eigenvalues of C^* matrix of residual design and is obtained as

$$\phi_1(s) = \frac{2}{s-1} + \frac{(s-3)}{s} = \frac{s^2-2s+3}{s(s-1)}. \tag{2.5}$$

Hence, using (2.4) and (2.5), we obtain the efficiency of the residual design d^*

$$e(s) = 1 - \frac{2}{[(s-1)^2+2]}, \text{ so the loss of efficiency is } \frac{2}{[(s-1)^2+2]} \tag{2.6}$$

□

Example 2.1. *Consider a standard Latin square design of size 7.*

1 2 3 4 5 6 7
2 3 4 5 6 7 1
3 4 5 6 7 1 2
4 5 6 7 1 2 3
5 6 7 1 2 3 4
6 7 1 2 3 4 5
7 1 2 3 4 5 6

Table 1: Overall A-efficiency of Latin square design when one pair of treatments is lost in any one row

Serial Number	Size of Latin square design (s)	Efficiency Factor $e(s)$
1	5	0.8889
2	6	0.9259
3	7	0.9474
4	8	0.9608
5	9	0.9697
6	10	0.9759
7	11	0.9804
8	12	0.9837

Now, let us consider a case, where treatment pair (1, 2) from row -1 *is lost. The residual design is*

$$\begin{array}{ccccccc} \boxed{1\ 2} & & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 5 & 6 & 7 & 1 \\ 3 & 4 & 5 & 6 & 7 & 1 & 2 \\ 4 & 5 & 6 & 7 & 1 & 2 & 3 \\ 5 & 6 & 7 & 1 & 2 & 3 & 4 \\ 6 & 7 & 1 & 2 & 3 & 4 & 5 \\ 7 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

Non zero eigenvalues of C^* *matrix of residual design are 6 and 7 with multiplicity 2 and 4 respectively, along with efficiency factor* $e(s) = 0.9474$.

Remark 2.1. *It can be concluded from the Table 1 that Latin square design is fairly robust against loss of one pair of treatments in any one row. Further, it is also observed that as size of the Latin square design increases, efficiency also increases.*

3 Robustness of Latin square design against loss of two pairs of treatments in any two rows

Consider a Latin square design of size s. Let two pairs of treatments are deleted from any two rows of this Latin square design. Call such design a residual design. Next we obtain the C^* matrix and nonzero eigen value of C^* matrix of residual design. Moreover we are interested to show that Latin square designs are fairly robust against the unavailability of a pair of treatments which is discussed in Theorem 3.1, 3.2 and 3.3 depending upon the situations.

3.1 Residual designs obtained from Latin square design

Consider a standard Latin square design d of size s. Now, if two pairs of treatments are lost from any two rows of a Latin square design, any one of the following three cases will occur:
Case-1: Number of common treatments lost in affected rows is zero. i.e., $c = 0$
Case-2: Number of common treatments lost in affected rows is one. i.e., $c = 1$
Case-3: Number of common treatments lost in affected rows is two. i.e., $c = 2$

Let us consider these cases one by one.

Case-1: Number of common treatments lost in affected rows is zero. i.e., $c = 0$

Let two pairs of distinct treatments are lost from any two rows of a standard Latin square design. Here, number of common treatments lost in both the affected rows, say c, is zero. Call this design as a residual design, which becomes an incomplete block design with two types of rows and two types of columns. The size of the two rows, where a pair of treatments is lost, is $(s-2)$, and the size of the remaining rows retain.

Next consider row as block. This incomplete block design has the parameters, $v = s$, $b = s$, $r_1 = s$ (for those treatments which are not lost), $r_2 = s - 1$ (for lost treatments), $k_1 = s$, $k_2 = s - 2$. For the residual design, pairs of treatments, say, $\lambda_1, \lambda_2, \lambda_3$ occur together in following three ways

1. $\lambda_1 = s$, for those treatments, which are not lost.
2. $\lambda_2 = s-1$, either for those treatments, which are lost, in the two affected rows or for (i, j) treatments, $(i \neq j)$, where i represents a treatment available among the lost treatments and j represents that treatment, which is not lost in the affected rows.
3. $\lambda_3 = s - 2$, for those lost treatments, which are present in two affected rows such that treatment i of one row associate with treatment j of another row and vice versa.

3.1.1 C^*-matrix and its non zero eigenvalues of residual design

C^*-matrix of the residual design can be expressed as

$$k(k-2)C^* = \begin{bmatrix} A & B & D \\ B' & A & E \\ D' & E' & F \end{bmatrix}_{s\times s} \tag{3.1}$$

where, $A = s(s-1)(s-2)I_2 - [(s-1)(s-2)+2]E_{22}$, $B = (s-2)^2E_{22}$,
$D = -[(s-1)(s-2)+2]E_{2(s-4)}$, $E = -(s^2-3s+4)E_{2(s-4)}$ and
$F = s^2(s-2)I_{(s-4)}[s(s-2)+4]E_{(s-4)(s-4)}$

The Nonzero eigenvalues of C^*-matrix of residual design d^* are (i) $(s-1)$ with multiplicity 2, (ii) s with multiplicity $(s-5)$ (iii) $\frac{(s-1)(s-2)+2}{(s-2)}$ with multiplicity 1 and (iv)

$$\frac{(s-1)(s-2)-2}{(s-2)} \text{ with multiplicity 1.} \tag{3.2}$$

Next in Theorem 3.1, we discuss how to obtain a robust design.

Theorem 3.1. *Latin square design of size s is fairly robust against the loss of two pairs of treatments in any two rows where number of common treatments lost is zero provided overall A-efficiency is given by*

$$e(s) = 1 - \frac{4(x+2s)}{[(s-1)^2+4]x+8s}, \quad \textit{where } x = [(s-1)^2(s-2)^2-4].$$

Proof. Consider a standard Latin Square Design of size s whose C-matrix and eigenvalues are shown in (2.1). Now, two pair of treatments is lost from any two row of this Latin square design and hence design becomes residual design whose C matrix and non zero eigen values are shown in (3.1) and (3.2). The overall A-efficiency of the residual design is obtained from

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{3.3}$$

where $\phi_2 s) = \frac{s-1}{s}$ and

$$\phi_1(s) = \frac{[s^2-2s+5][(s-1)^2(s-2)^2-4]+8s}{s(s-1)[(s-1)^2(s-2)^2-4]}. \tag{3.4}$$

Hence, using (3.3) and (3.4) and after solving, we obtain the efficiency of the residual design d^* as

$$e(s) = \frac{(s-1)^2 x}{[(s-1)^2+4]x+8s}, \quad \text{where } x = [(s-1)^2(s-2)^2-4]$$

In term of loss of efficiency, $e(s)$ is further expressed as

$$e(s) = 1 - \frac{4(x+2s)}{[(s-1)^2+4]x+8s}, \text{ so loss of efficiency is } \frac{4(x+2s)}{[(s-1)^2+4]x+8s} \tag{3.5}$$

$\square$

Example 3.1. *Consider a standard Latin square design of size 7 shown in example 2.1. Now, let us consider a case, where treatment pair (1, 2) from row* -1 *and treatment pair (3, 4) from row* -2 *are lost. The residual design is*

$$\begin{array}{ccccccc}
\boxed{1\ 2} & & 3 & 4 & 5 & 6 & 7 \\
2 & \boxed{3\ 4} & & 5 & 6 & 7 & 1 \\
3 & 4 & 5 & 6 & 7 & 1 & 2 \\
4 & 5 & 6 & 7 & 1 & 2 & 3 \\
5 & 6 & 7 & 1 & 2 & 3 & 4 \\
6 & 7 & 1 & 2 & 3 & 4 & 5 \\
7 & 1 & 2 & 3 & 4 & 5 & 6
\end{array}$$

The nonzero eigenvalues of C^* matrix of residual design are 6, 7, 32/5 and 28/5 with multiplicity 2, 2, 1, and 1 respectively, along with the efficiency factor $e(s) = 0.8986$.

Case 2. Number of common treatments lost in affected rows is one, i.e., $c = 1$

Let two pairs of treatments are lost from any two rows of a standard Latin square design, such that number of common treatments lost in both the affected rows, say c, is one. Call this design as a residual design and assume it a connected design. The residual design becomes an incomplete block design with two types of rows and two types of columns. The size of the two rows, where a pair of treatments is lost, is $(s-2)$, and the size of the remaining rows retain. Similarly the size of the columns will be unequal.

Next, consider row as block. This incomplete block design has the parameters, $v = s$, $b = s$, $r_1 = s$ (for those treatments which are not lost), $r_2 = s-1$ (for those treatments which are lost once), $r_3 = s-2$ (for those treatments which are lost twice), $k_1 = s$, $k_2 = s-2$. For the residual design, pair of treatments, say, $\lambda_1, \lambda_2, \lambda_3$ occurs together in following three ways

1. $\lambda_1 = s$, for those treatments, which are not lost.

2. $\lambda_2 = s-1$, for those (i, j) treatments, $(i \neq j)$, where i represents a treatment lost once in any of the affected rows and j represents those treatments, which are present (except the lost treatments as a pair of treatments), in any of the two affected rows.

3. $\lambda_3 = s-2$, for either of the cases:

   (a) For (i, j) treatments, $(i \neq j)$, where i and j both represent treatments lost once in the affected rows.

   (b) For (i, j) treatments, $(i \neq j)$, where i represents a treatment available among the treatments lost once and j represents that treatment, which is lost twice in the affected rows.

   (c) For (i, j) treatments, $(i \neq j)$, where i represents a treatment lost twice and j represents those treatments, which are present (except the lost treatments) in any of the two affected rows.

3.2 C^*-matrix and nonzero eigenvalues of residual design

C^*-matrix of the residual design is expressed as

$$k(k-2)C^* = \begin{bmatrix} A & B & D \\ B' & F & X \\ D' & X' & Y \end{bmatrix}_{S\times S} \tag{3.6}$$

where, A is a 2×2 sub matrix and expressed as
$A = s(s-2)(s-3)I_2 - (s-2)^2E_{22}$, B is a 2×1 column vector and expressed as $B = -(s-2)^2E_{21}$, D is a $2\times(s-3)$ sub matrix as $D = -[(s-1)(s-2)+2]E_{2\times(s-3)}$, F is a scalar as $F = (s-1)(s-2)^2$, X is a $1\times(s-3)$ row vector as $X = -(s-2)^2E_{1\times(s-3)}$ and Y is a square symmetric sub-matrix of order $(s-3)\times(s-3)$ expressed as $Y = s^2(s-2)I_{(s-3)} - [s(s-2)+4]E_{(s-3)\times(s-3)}$.

From (3.6), we obtain the nonzero eigenvalues of C^*-matrix of residual design which are following: (i) $[(s-2)(s^2-s-1)]/s(s-2)$ with multiplicity 1 (ii) $(s-2)$ with multiplicity 1 (iii) s with multiplicity $(s-4)$ and (iv)

$$\frac{(s-2)(s^2-s+1)+2}{s(s-2)} \text{ with multiplicity 1.} \tag{3.7}$$

Theorem 3.2. *Latin square design of size s is fairly robust against the loss of two pairs of treatments in any two rows when common treatments lost is one provided overall A-efficiency is given by*

$$e(s) = \frac{4s^2 - 26s^3 + 62s^2 - 62s + 18}{s^6 - 9s^5 + 37s^4 - 89s^3 + 127s^2 - 95s + 24}.$$

Proof. Consider a standard Latin square design of size s. Since two pairs of treatment are lost in two rows and hence design does not remain LSD, call this design a residual design. Now We obtain the overall A-efficiency of the residual design from, $e(s) = \frac{\phi_2(s)}{\phi_1(s)}$ where, $\phi_2(s) = \frac{s-1}{s}$, and

$$\phi_1(s) = \frac{s(s-2)}{(s-2)(s^2-s-1)-2} + \frac{1}{s-2} + \frac{(s-4)}{s} + \frac{s(s-2)}{(s-2)(s^2-s+1)+2}.$$

Now using $e(s) = \frac{\phi_2(s)}{\phi_1(s)}$ and after solving we obtain the efficiency of the residual design d^* as,

$$E(s) = 1 - \frac{4s^2 - 26s^3 + 62s^2 - 62s + 18}{s^6 - 9s^5 + 37s^4 - 89s^3 + 127s^2 + 24}$$

□

Example 3.2. *Consider a standard Latin square design of size 7 shown in Example 2.1. Now, let us consider a case, where treatment pair (1, 3) from row* -1 *and treatment pair (2, 3) from row* -2 *are lost. The residual design is shown below*

$$
\begin{array}{ccccccc}
\boxed{1} & 2 & \boxed{3} & 4 & 5 & 6 & 7 \\
\multicolumn{2}{c}{\boxed{2\ 3}} & 4 & 5 & 6 & 7 & 1 \\
3 & 4 & 5 & 6 & 7 & 1 & 2 \\
4 & 5 & 6 & 7 & 1 & 2 & 3 \\
5 & 6 & 7 & 1 & 2 & 3 & 4 \\
6 & 7 & 1 & 2 & 3 & 4 & 5 \\
7 & 1 & 2 & 3 & 4 & 5 & 6
\end{array}
$$

The non zero eigenvalues of C^* matrix of the residual design are 5, 203/35, 217/35 and 7 with multiplicity 1, 1, 1, 3 respectively, along with efficiency factor $e(s) = 0.8907$.

Case 3. Number of common treatments lost in affected rows is two i.e., $c = 2$

Let two same pairs of treatments are lost from any two rows of a standard Latin square design. Here, number of common treatments lost in both the affected rows, say, c is two. Call this design as a residual design. The residual design becomes an incomplete block design with two types of rows and two types of columns. The size of the two rows, where a pair of treatments is lost, is $(s-2)$, and the size of the remaining rows retain.

Next, consider row as block. This incomplete block design has the parameters, $v = s$, $b = s$, $r_1 = s$ (for those treatments which are not lost), $r_2 = s-2$ (for lost treatments), $k_1 = s$, $k_2 = s-2$. For the residual design, pair of treatments, say, λ_1, λ_2 occurs together in following two ways

1. $\lambda_1 = s$, for those treatments, which are not lost.
2. $\lambda_2 = s-2$, for either of the cases
   (a) For those treatments, lost in the affected rows.
   (b) For (i, j) treatments, $(i \neq j)$, where i represents a treatment, available among the lost treatments and j represents that treatment, which is commonly present, but not lost in the affected rows.

3.3 C^*-matrix and nonzero eigenvalues of the residual design

For case 3, the C^* matrix and the non zero eigenvalues of the residual are obtained as

$$k(k-2)C^* = \begin{bmatrix} A & D \\ D' & B \end{bmatrix}_{s\times s} \tag{3.8}$$

where, A is a 2×2 sub-matrix and is expressed as

$$A = s(s-2)^2 I_2 - (s-2)^2 E_{22},$$

B is a square symmetric sub-matrix of order $(s-2)\times(s-2)$ and is expressed as

$$B = s^2(s-2)I_{(s-2)} - [s(s-2)+4]E_{(s-2)\times(s-2)},$$

D is a $2\times(s-2)$ sub-matrix and is expressed as

$$D = -(s-2)^2E_{(s-2)\times(s-2)}.$$

Using (3.8), we obtained the nonzero eigenvalues of C^*-matrix of residual design which are $(s-2)$ and s with multiplicity 2 and $(s-3)$ respectively. Next we have following Theorem.

Theorem 3.3. *Latin square design of size* s *is fairly robust against the loss of two pairs of treatments in any two rows when number of common treatments lost is two, provided overall* A*-efficiency is given by*

$$e(s) = 1 - \frac{4}{(s-1)(s-2)+4}.$$

Proof. Consider a standard Latin Square Design of size s. Since two pair of treatments are lost in two rows and hence design does not remain LSD, call this design a residual design. Now We obtain the overall A-efficiency of the residual design from,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{3.9}$$

Here $\phi_2(s)$ and $\phi_1(s)$ are obtained as $\phi_2(s) = \frac{s-1}{s}$, and

$$\phi_1(s) = \frac{(s-1)(s-2)+4}{(s-1)(s-2)} \tag{3.10}$$

Hence, using (3.9) and (3.10), we obtain the efficiency of the residual design d^* as,

$$E(s) = \frac{(s-1)(s-2)}{(s-1)(s-2)+4} \tag{3.11}$$

□

Example 3.3. *Consider a standard Latin square design of size 7, shown in Example 2.1. Now, let us consider a case, where treatment pair (6,7) from row-1 and treatment pair (6,7) from row-2 are lost. The residual design is*

$$\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & \boxed{6\ 7} \\
2 & 3 & 4 & 5 & \boxed{6\ 7} & 1 \\
3 & 4 & 5 & 6 & 7 & 1 & 2 \\
4 & 5 & 6 & 7 & 1 & 2 & 3 \\
5 & 6 & 7 & 1 & 2 & 3 & 4 \\
6 & 7 & 1 & 2 & 3 & 4 & 5 \\
7 & 1 & 2 & 3 & 4 & 5 & 6
\end{array}$$

Table 2: Overall A-efficiency of Latin square design when two pairs of treatments are lost in any two rows

Sr. No.	Size of Latin square design	Efficiency Factor case 1	case 2	case 3
1	5	0.7887	0.7716	0.7500
2	6	0.8585	0.8466	0.8333
3	7	0.8986	0.8907	0.8824
4	8	0.9239	0.9186	0.9130
5	9	0.9409	0.9372	0.9333
6	10	0.9528	0.9501	0.9474
7	11	0.9614	0.9595	0.9574
8	12	0.9679	0.9664	0.9649

For this residual design the nonzero eigenvalues of C^* matrix are 5 and 7 with multiplicity 2 and 4 respectively, along with efficiency factor $e(s) = 0.8824$

4 Lower and upper bound of overall A-efficiency of Latin square design when two pairs of treatments are lost in any two

Here we obtained the lower bound and upper bound of overall A-efficiency of Latin square design, when two pairs of treatments are lost in any two rows, as following:

Lower bound of overall A-efficiency is $1 - \frac{4}{(s-1)(s-2)+4}$;

Upper bound of overall A-efficiency is $1 - \frac{4(x+2s)}{[(s-1)^2+4]x+8s}$,

where $x = [(s-1)^2(s-2)^2 - 4]$.

5 Conclusion

It can be concluded from the Table 1 that Latin square design is fairly robust against loss of one pair of treatments in any one row. Further, it is also observed that as size of the Latin square design increases, efficiency also increases. From Table 1, it is observed that efficiency decreases as number of lost pair of treatments increases.

Similarly, it is also noticed from the Table 2 that Latin square designs of size six and greater than six are robust against loss of two pairs of treatments in any two rows. Further, it is also observed that,

1. As size of the Latin square design increases, efficiency increases.

2. As number of lost common treatments increases, efficiency decreases. i.e., for any Latin square design of size s,

$$e(s_0) < e(s_1) < e(s_2)$$

where, $e(s_0)$ denotes overall A-efficiency of Latin square design against loss of two treatment pairs for case-1 (number of common treatments lost is zero. i.e., $c = 0$), $e(s_1)$ denotes overall A-efficiency of Latin square design against loss of two treatment pairs for case-2 (number of common treatments lost is one. i.e., $c = 1$), and $e(s_2)$ denotes overall A-efficiency of Latin square design against loss of two treatment pairs for case-3 (number of common treatments lost is two. i.e., $c = 2$).

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