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Design of Single Sampling Plan by Discriminant at MAPD

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Abstract. This paper presents a procedure for designing a single sampling plan indexed through maximum allowable proportion defective (MAPD) along with Tangential distance (Discriminant). It is discussed how the declination angle of the tangent at the inflection point of the OC curve discriminates the SSP. Tables are presented for the selection of plans based on MAPD with Discriminant or declination angle of the tangent. Also an optimum sampling plan is introduced for an Operating Ratio of MAPD to Discriminant and the uses were studied.
Keywords Single Sampling Plan, Maximum Allowable Proportional Defective, Tangent intercept, Inflection point, Operating Characteristic curve, Discriminant, Operating Ratio and Declination angle. Optimum Sampling Plan.

1 Introduction

MAPD is a key measure assessing to what degree the inflection point empowers the OC curve to discriminate between good and bad lots. Mayor [3] introduced the concept of MAPD in a SSP using Poisson model. MAPD locates a point of the OC curve at which the descent is steepest. It is defined as the proportion of defective beyond which consumer won't be willing to accept the lot. Mandelson [2] has explained the desirability for developing a system of sampling plan indexed by MAPD and suggested a relation $p^* = c/n$. Soundararajan [5] has indexed SSP through MAPD and $K(p_T/p^*)$. Muthuraj and Soundararajan [4]has studied the selection of single sampling plan indexed by p^* with its relative slope. Norman Bush [1]developed a SSP based on tangent at inflection point. This technique states that a straight line will uniquely portray the slope of OC curve and hence the OC is fixed. According to Norman Bush the point of inflection has been chosen because it is most representative point of an OC curve showing the turning point of quality since the maximum tangent occurs at this point. Ramkumar [6] has developed a selection procedure for

SSP involving AQL and tangent intercept in the p axis. Ramkumar [7] developed a set of sampling plans indexed through MAAOQ, MAPD explaining the adequacy of MAPD as quality.

A tangent is drawn at the inflection point of an OC curve, where p_T is the tangent intercept to the p-axis and $L(p_T)$ is the tangent intercept to the $L(p)$-axis. Define Discriminant $\mathbf{D}$ as the distance in the $L(p)$ coordinate of the inflection point, from $L(p^*)$ to the point $L(p_T)$, i.e. $D = L(p_T) - L(p^*)$. When $\mathbf{D}$ is large there is a high degree of discrimination power and when D is small the discrimination decreases (From Figure 1). The Discriminant $\mathbf{D}$ is a function of c alone (Equation: 6) and it is monotonically increasing with respect to c (Table 1). Therefore there exists an acceptance number c corresponding to each value of D i.e. there is a unique sampling plan for each D. The slope of the OC curve is horizontal at the inflection point and a sharp decline in the probability of acceptance is expected for the proportion of defective beyond p^*. The declination angle A of the tangent also represents the discrimination power of the OC curve. When A decreases OC curve becomes more stringent. From the relation Tan $A = p^*/\mathbf{D}$ one can find (n, c) for a given p^* and D. Thus it is seen that both A and $\mathbf{D}$ uniquely determines the SSP.

2 Selection of sampling plans

$\mathbf{D}$ is given for twenty values of $c \geq 1$ in Table 1. For a given value of $\mathbf{D}$ the value of c can be easily read off from the Table 1 (to the next higher value of $\mathbf{D}$). SSP with $c = 0$ are not considered, since $c = 0$ plans do not have an inflection point on the OC curve.

1) For a specified $\mathbf{D}$ and p^*

$\mathbf{D}$ is a monotonic increasing function of c, there exists a unique c for a given $\mathbf{D}$ and then $n = c/p^*$, where given $\mathbf{D}$ is fixed nearer to the next defined $\mathbf{D}$. Table 1 gives the value of $L(p_T)$, $L(p^*)$ and $\mathbf{D}$ for $c = 1, 2, \ldots, 20$.

Example 2.1 *A manufacturer of plastic products fixes the maximum allowable proportion defectives at 13% and Discriminant of the OC curve* $\mathbf{D} = 1.185$.

From Table 1, for a given value of $\mathbf{D} = 1.185$ *(*> 1.1167 *and* < 1.1858*),* $c = 8$ *and* $n = 61.54 = 62$.

Also the angle of declination for (62, 8) Tan $A = 0.1097$ *and* $A = 6.28^\circ$. *(This can be easily found from Table 3)*

2) For a specified angle A and p^*

Find $\mathbf{D}$ from the relation $\mathbf{D} = p^*/\tan A$. Then it is easy to fix the corresponding c and hence n using Table 1. It is clear that $\mathbf{D}$ is inversely proportional to angle A for fixed p^*, implies that more efficient OC curves can be developed based on angle A.

Example 2.2 *A company claims that the OC curve for their electrical switches has MAPD= 5% and angle of declination of tangent at MAPD is 5°,* $\mathbf{D} = p^*/\tan A$ *i.e.* $\mathbf{D} = 0.05/0.0875 = 0.5714$

From Table: 1, for $\mathbf{D} = 0.5714$ (> 0.5413 with $c = 2$) and $p^* = 0.05$, the sampling plan is $(40, 2)$.

3) For a specified $\mathbf{D}$ and A

Table 3 are constructed to find suitable sampling plans when $\mathbf{D}$ and A are specified. Trace the angle A (numerically exceeding) in Table 3 and find out the corresponding p^* at A for given $\mathbf{D}$. Correspondingly (n, c) is obtained.

Example 2.3 *In an OC curve to which a tangent is drawn through the inflection point has* $\mathbf{D} = 0.8773$ *and declination angle A =8°.*
Since $\tan A = p^*/D$ $p^* = \boldsymbol{D}$ *Tan A, i.e.* $p^* = 0.12$
From the table 3 $n = 42$ *and* $c = 5$.
Then the value of AQL and LTPD for the SSP (42, 5) can be determined using conversion table, see Ramkumar [6] Thus $p_1 = 0.0622$ *and* $p_2 = 0.2208$*, i.e. there is a variation of 6% to 22% in AQL-LTPD values.*

Consider the sampling plans, where AQL is fixed:
If $p_1 = 0.0622$ and $p_2 = 0.25$, the corresponding SSP will be (32, 4).
If $p_1 = 0.0622$ and $p_2 = 0.20$, the corresponding SSP will be (53, 6)
OC curves for the SSP (42, 5), (32, 4) and (53, 6) are given in Figure 2.

Comparing the OC curves it is seen that for lesser LTPD value than that obtained from tangential angle plan the better quality products are rejected. For greater LTPD values, unwanted products will be accepted. Implying that tangential angle plan is more efficient for distinguishing good and bad items reasonably at acceptable proportion defective. Discriminant can be defined as a cost function and appropriate sampling plan can be designed for given MAPD. Practically $\mathbf{D}$ is equated to the cost and OC curve is fixed at the requirement by finding suitable MAPD.

Practically $\mathbf{D}$ can be defined as a cost function of a unit and then corresponding sampling plans are evaluated. As the cost increases an effective OC curve can be explained.

Theorem 2.1 *The discriminant* $\mathbf{D}$ *is independent of sample size* n.

By definition the discriminant $\mathbf{D}$ is the distance from the ordinate of the inflection point $(L(p^*))$ to the tangent intercept $(L(p_T))$ in the $L(p)$ axis . We have

$$\mathbf{D} = L(p_T) - L(p^*) = \frac{e^{-c}c^{c+1}}{c!},$$

which is a function of c alone (see Figure 3).
Implies that $\mathbf{D}$ is independent of sample size n and there exist a unique constant value of $\mathbf{D}$ for each c.

3 Construction of tables

The probability of acceptance of a lot with quality p is given by the Poisson model

$$L(p) := e^{-np} \sum_{r=0}^{c} \frac{(n \cdot p)^r}{r!} \tag{3.1}$$

where p is the proportion defective of the lot, p coordinate of the inflection point will be obtained as

$$p^* = c/n \tag{3.2}$$

The tangent intercept at p-axis is given by

$$\mathrm{PT} := \frac{c}{n} + \frac{c!}{c^c \cdot n} \sum_{r=0}^{c} \frac{c^r}{r!} \tag{3.3}$$

Therefore the tangent intercept at $L(p)$-axis, $L(p_T)$ is given as

$$L(\mathrm{PT}) := \frac{e^{-c} c^{c+1}}{c!} + e^{-c} \cdot \sum_{r=0}^{c} \frac{c^r}{r!} \tag{3.4}$$

$L(p^*)$ represents the probability of acceptance of an utmost satisfactory quality (MAPD)

$$L(p^*) = e^{-c} \cdot \sum_{r=0}^{c} \frac{c^r}{r!} \tag{3.5}$$

Then the Discriminant $\mathbf{D} = L(p_T) - L(p^*)$.

$$\mathbf{D} = \frac{e^{-c} \cdot c^{c+1}}{c!} \tag{3.6}$$

Thus $\mathbf{D}$ is a function of c alone, and it is constant for fixed c (see Figure 3). From Figure 1

$$\tan A = p^*/\mathbf{D} \tag{3.7}$$

The declination angle

$$A = \tan^{-1}(p^*/\mathbf{D}) \tag{3.8}$$

$$D/np^* = \frac{e^{-c} c^c}{c!} \tag{3.9}$$

For different values of $c = 1, 2, \ldots, 20$, $\mathbf{D}$ is determined from equation (3.6). $L(p^*)$ and $L(p_T)$ can also be found for different c from equation (3.4) and (3.5). Prefixing c and hence $\mathbf{D}$, n is determined. Finding c from $\mathbf{D}$ and substituting c and p^* in equation (3.9), n is determined. Similarly substituting p^* for fixed $\mathbf{D}$ in equation (3.8) angle A can be determined.

4 Optimum Sampling Plan

For various values of MAPD and Discriminant, there exist a set of sampling plans suitable for defined qualities. But there exist a unique sampling plan with maximum efficiency of the OC curve. Such OC curve having minimum c and maximum discriminatory power develops a sampling plan and it is called Optimum sampling plan. Thus for a particular value of Operating Ratio = MAPD/ Discriminant (which will be ultimately Tan A), a collection of sampling plans can be developed in a specific range of sample size. The first intersection of the value of Operating Ratio in the stream of sampling plans will be the optimum sampling plan with the stringent OC having minimum c among the others. Such a sampling plan contains better quality accepted comparing with all other plans with the defined OR.

Example 4.1 *A company fix a primary quality index on OR = .0833 and it is planned to use sampling plans with utmost stringency at the initial lots and eager to switch on to successive levels on continuous acceptance. What will be the minimal security plan and maximum security plan?*

Fixing OR at a constant with an admissible range of sampling inspection $n = 50 to 100$ from the fig 4(a) possible sampling plans are: (54, 3) (62, 4) (69, 5) (75, 6) (81, 7) (86, 8) (92, 9) and (96, 10).

In these sampling plans the optimum sampling plans are (54, 3) and (96, 10). First intersection (54, 3) is stringent and last intersection (96, 10) is producer friendly.

5 Significance of optimum plan

For regularly consumable homely products with moderate quality like electrical goods, utensils, plastic goods, *etc. the quality can be defined in terms of MAPD. But prefixing a quality level at the elementary stage of production is difficult. In these occasions one can begin the production on an Operating Ratio like MAPD/***D** *which entertains both consumer's condition on quality and producer's risk in terms of sharpness of the OC curve. Also it estimates sample size from the range of sample that can be accommodated by the producer for inspection and making use of all these elements a sampling plan is prepared. So one can confidently produce the items in the beginning stages with such a sampling plan and the consumers can propose further refinement on MAPD or sharpness or sample size. The switching rules can be implemented at successive stages of inspection according to the requirement.*

6 Figures and Tables

Figure 1 depicts the Discriminant $\mathbf{D}$*, declination angle* A *for SSP (50, 2) where* $P_a(p)$ *represents the OC curve,* $L1(p)$ *represents the tangent and* $\mathbf{D} = L(p_T) - L(p^*)$*.*
Figure 2 show the efficiency of tangential angle plan for attaining most appropriate OC curve.
Figure 3 shows that the discriminant is unaffected by sample number and it is a constant for fixed c*. Also, it shows that the OC curve is more stringent when* $\tan A$*, the inflection tangential angle decreases. Also it ascertains that* $\mathbf{D}$ *is a constant for fixed* c*.*
Figure 4(a) represent the OR ($\tan A$*) for distinct values of* c *for the range of* n *varying from 50–100, from which the possible sampling plans for the specified value of OR can be detected. Similarly one can draw OR for different ranges of* n*. Figure 4(b) ranges* n *over 100 to 250 and Figure 4(c) over 250 to 500.*
Figure 5 is the comparison of OC curves indicating the discriminating power of the OC curve. It is used to establish that the Optimum Sampling Plan is determined as one with minimum c *or with maximum* c *as the case is required.*
Table 1 shows the Discriminant (using (3.6)*) and acceptance number can be identified either specifying* $\mathbf{D}$ *or* $L(p^*)$ *or* $L(p_T)$ *for* $c = 1, 2, \ldots, 20$*.*
Table 2 shows the sampling plans for different values of MAPD and Discriminant $\mathbf{D}$*.*
Table 3 shows the sampling plans for different values of MAPD and Discriminant $\mathbf{D}$ *and Tangential Angle* A*, provided any two of them are available.*

For Figure 2

$Pa(p)$	c	n	p^*	$\mathbf{D}$	$\tan A$
$P_{1a}(p)$	5	42	0.119048	0.877337	0.135692
$P_{2a}(p)$	4	32	0.125	0.781467	0.159956
$P_{3a}(p)$	6	53	0.113208	0.963739	0.117467

From Figure 2 the OC Curve is more reasonable with parameters $\mathbf{D}$ *and* A *compared to AQL and LTPD.*

For Figure 3, $c = 2$*,* $n = 50, 100$ *thus* $p^* = .04$ *and* $.02$ *respectively. The inflection tangent meets at 1.218 and* $Pa(p^*) = 0.677$ *so that* $D = 0.541$*.*

7 Switching Rule.

For a fixed OR, find plausible sampling plans within a fixed range.
These plausible sampling plans will lead to a switching rule.
Start with moderate (n, c)*, fixing at middle falling OC curve.*
Continue the inspection till k *lots are inspected (*k *depends on the requirement of quality).*

Table 1: The Discriminant, y-intercept of MAPD & Tangent intercept for $c = 1, 2, \ldots, 20$

c	$\mathbf{D}$	$\mathbf{D}/np^*$	$L(P^*)$	$L(P_T)$	c	$\mathbf{D}$	$\mathbf{D}/np^*$	$L(P^*)$	$L(P_T)$
1	0.3679	0.368	0.7358	1.1037	11	1.3132	0.119	0.5793	1.8925
2	0.5413	0.271	0.6767	1.218	12	1.3724	0.114	0.5759	1.9483
3	0.6721	0.224	0.6472	1.3193	13	1.4292	0.110	0.5731	2.0023
4	0.7815	0.195	0.6289	1.4104	14	1.4838	0.106	0.5704	2.0542
5	0.8773	0.175	0.6159	1.4932	15	1.5365	0.102	0.5681	2.1046
6	0.9637	0.161	0.6063	1.57	16	1.5875	0.099	0.5659	2.1534
7	1.0430	0.149	0.5987	1.6417	17	1.6368	0.096	0.5640	2.2008
8	1.1167	0.140	0.5926	1.7093	18	1.6848	0.094	0.5623	2.2471
9	1.1858	0.132	0.5874	1.7732	19	1.7315	0.091	0.5606	2.2921
10	1.2511	0.125	0.5830	1.8341	20	1.7767	0.089	0.5591	2.3358

Table 2: Certain SSP for given Discriminant $\mathbf{D}$ and MAPD

c	$p^* : -$	.05	.08	.10	.12	.14	.15	.18	.20
	$\mathbf{D}$	n	n	n	n	n	n	n	n
1	0.3679	20	13	10	8	7	7	6	5
2	0.5413	40	25	20	17	14	13	11	10
3	0.6721	60	38	30	25	21	20	17	15
4	0.7815	80	50	40	33	29	27	22	20
5	0.8773	100	63	50	42	36	33	28	25
6	0.9637	120	75	60	50	43	40	33	30
7	1.0430	140	88	70	58	50	47	39	35
8	1.1167	160	100	80	67	57	53	44	40
9	1.1858	180	113	90	75	64	60	50	45
10	1.2511	200	125	100	83	71	67	56	50
11	1.3132	220	138	110	92	79	73	61	55
12	1.3724	240	150	120	100	86	80	67	60
13	1.4292	260	163	130	108	93	87	72	65
14	1.4838	280	175	140	117	100	93	78	70
15	1.5365	300	188	150	125	107	100	83	75
16	1.5875	320	200	160	133	114	107	89	80
17	1.6368	340	213	170	142	121	113	94	85
18	1.6848	360	225	180	150	129	120	100	90
19	1.7315	380	238	190	158	136	127	106	95
20	1.7767	400	250	200	167	143	133	111	100

Table 3: Certain SSP for given **D** and angle A

c	**D**	P^* : −	.01	.05	.08	.11	.13	.15	.20	
1	0.3679		A = 1.55699	7.7395	12.2680	15.2064	16.6464	19.4612	22.1817	28.5297
			n=100	20	13	10	9	8	7	5
2	0.5413		1.0584	5.2775	8.4070	10.4668	11.4869	13.5045	15.4886	20.2783
			200	40	25	18	15	13	10	
3	0.6721		.8524	4.2546	6.78797	8.4628	9.29497	10.9472	12.5812	16.5717
			300	60	38	27	23	20	15	
4	0.7815		.7331	3.6608	5.8449	7.2919	8.0120	9.4445	10.8651	14.3549
			400	80	50	37	31	27	20	
5	0.8773		.6531	3.2619	5.2103	6.5029	7.1467	8.4289	9.7026	12.8424
			500	100	63	46	39	33	25	
6	0.9637		.5945	2.9700	4.7454	5.9242	6.5118	7.6826	8.8471	11.7244
			600	120	75	55	46	47	30	
7	1.0430		.5493	2.7446	4.3861	5.4766	6.0204	7.1047	8.1839	10.85496
			700	140	88	64	54	47	35	
8	1.1167		.5131	2.5637	4.0977	5.1172	5.6257	6.6402	7.6504	10.15397
			800	160	100	73	62	53	40	
9	1.1858		.4832	2.4145	3.8596	4.8204	5.2998	6.2564	7.2095	9.5736
			900	180	113	82	69	60	45	
10	1.2511		.45795	2.2886	3.6587	4.5699	5.0247	5.9322	6.8368	9.0824
			1000	200	125	91	77	67	50	
11	1.3132		.4363	2.1805	3.4861	4.3547	4.7882	5.6536	6.5164	8.6596
			1100	220	138	100	85	73	55	
12	1.3724		.4175	2.0865	3.3361	4.1675	4.5826	5.4112	6.2375	8.2914
			1200	240	150	110	92	80	60	
13	1.4292		.4009	2.0037	3.2038	4.0024	4.4012	5.1973	5.9915	7.9662
			1200	260	163	119	100	87	65	
14	1.4838		.3861	1.92998	3.0862	3.8556	4.2398	5.0071	5.7725	7.6766
			1300	280	175	128	108	93	70	
15	1.5365		.3729	1.8638	2.9805	3.7237	4.0949	4.8362	5.5758	7.4163
			1500	300	188	137	116	100	75	
16	1.5875		.3609	1.80399	2.8849	3.6044	3.9638	4.6815	5.3978	7.1805
			1600	320	200	146	123	107	80	
17	1.6368		.3500	1.7497	2.7982	3.4961	3.8447	4.5411	5.2361	6.9664
			1700	340	213	155	131	113	85	
18	1.6848		.3401	1.69987	2.7186	3.3968	3.7355	4.4122	5.0877	6.7698
			1800	360	225	164	139	120	90	
19	1.7315		.3309	1.6541	2.6453	3.3054	3.6350	4.2937	4.9512	6.5889
			1900	380	238	173	146	127	95	
20	1.7767		.3225	1.61199	2.5781	3.2214	3.5428	4.1848	4.8258	6.4227
			2000	400	250	182	154	133	100	

Figure 1: The OC curve, Discriminant, tangent intercept and Tan A of a SSP (50, 2)

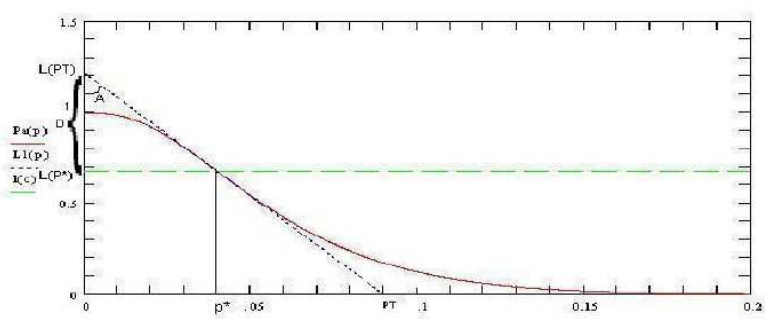


Figure 2: OC curves for the SSP (1) (42, 5), (2) (32, 4) and (3) (53, 6)

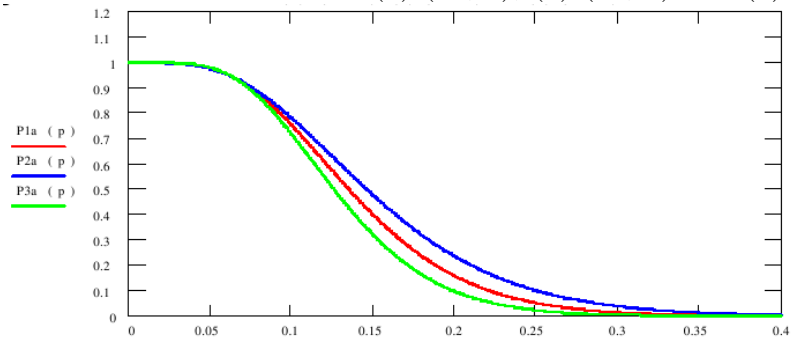
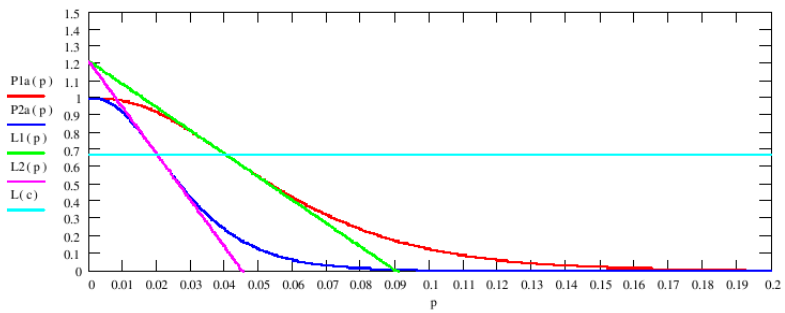
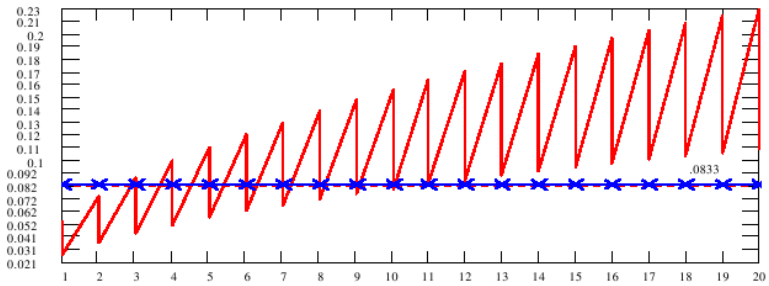


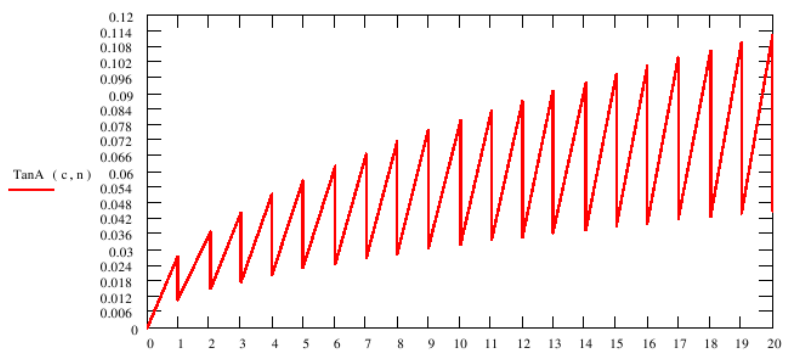
Figure 3: OC curves for SSP showing **D** is a constant for fixed c



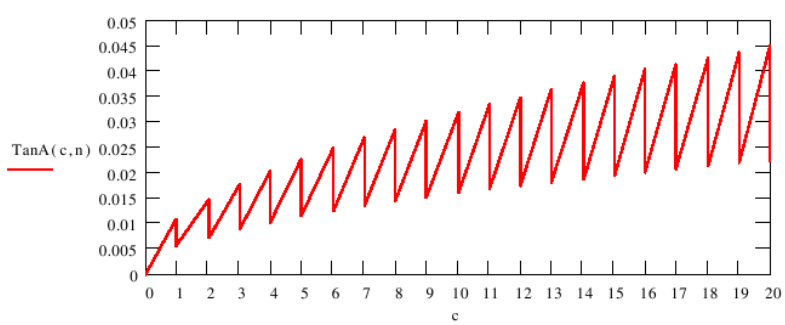
If no lots are rejected switch to next SSP and so on till maximum c is attained.
If at any stage the lot is rejected switch over to previous SSP and inspect $2k$ *lots.*
Allow tightening till minimum c is attained.
In example 2.4 possible sampling plans are: (54, 3) (62, 4) (69, 5) (75, 6) (81, 7) (86, 8) (92, 9) and (96, 10). Fix repetitive lot number $k = 10$.
For example start with (81, 7).
Repeat in 10 lots, if all are good, go to (86, 8).
Continue till, (92, 9), (96, 10) and coninue inspection till a rejection is occurred.
If one of them in 10 lots is not satisfactory at initial inspection level.
At the time of rejection go to (75, 6).
Continue in 20 lots.
If all are good go to (81, 7)
If one is again found bad in these lots switch backward to (69, 5)
Repeat the process till reaching (54, 3) and it is rejected.
This switching rule will lead to a stochastic process of reaching to optimum sampling plans.



(a) Graphical representation of $\tan A$ for $c = 1, 2, \ldots, 20$ and $n = 50, \ldots, 100$ the possible sampling plans with OR=0.0833



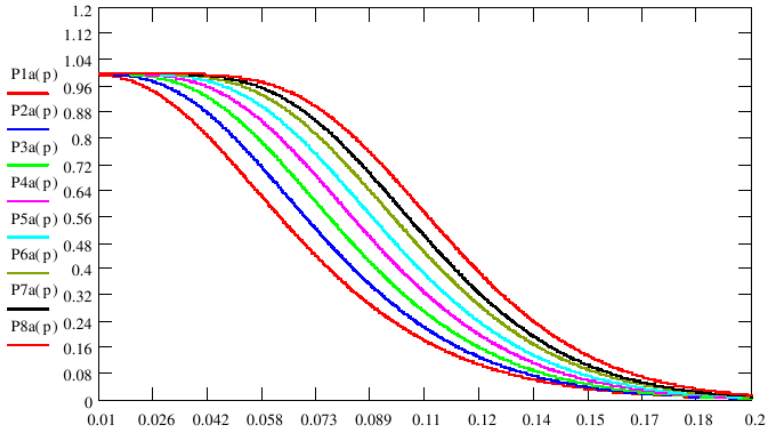
(b) Graphical representation of $\tan A$ for $c = 1, 2, \ldots, 20$ and $n = 100, \ldots, 250$



(c) Graphical representation of $\tan A$ for $c = 1, 2, \ldots, 20$ and $n = 250, \ldots, 500$

Figure 4:

Figure 5: OC curves based on OR=0.0833



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