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Revised multi-segment goal programming and applications

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Abstract. Liao [C.-N. Liao, Formulating the multi-segment goal programming, Computers & Industrial Engineering, 56, (2009)[9] 138–141] has proposed a new approach namely multi-segment goal programming (MSGP) for multi-objective decision making problems. However, to express the condition of multi-segment aspiration levels (MSAL) may exist in many marketing or decision management problems. In this paper, we proposed an alternative method to solve the MSGP problems with two contributions: (1) the alternative method represents a linear from of MSGP which can easily be solved by common linear programming sties, and (2) the alternative approach does not involve multiplicative terms of binary variables, this leads to more efficient use of MSGP and is easily understood by industrial participants. In addition, two examples compare with GP and multi-segment GP, and the practice application to marketing strategy are included to demonstrate the solution procedure of the proposed model. Finally, to demonstrate the usefulness of the proposed method, two illustrate example are included.

1. Introduction

Decision-making is part of our daily lives. Almost all decision/management problems have multiple and often conflicting criteria for solutions. A decision problem D is composed of $D = \langle A, Q, E, G, W\rangle$ under time horizon $[t_0, t_T]$, where a set of alternatives, $A = \{a_i|i = 1, \ldots, m\}$, should be evaluated by a decision maker (DM) under his/her preference structure, $W = \{w_k|k = 1, \ldots, K\}$, with respect to a set of criteria, $G = \{g_k|k = 1, \ldots, K\}$, under certain set of external factors, $E = \{e_p|p = 1, \ldots, P\}$, with its possible outcomes $O^{(i)} = \{O_j^{(i)}|j = 1, \ldots, N\}$. Accordingly, multi-criteria decision analysis is a general term that contains two types of problems as multi-attribute decision analysis, $D^* = \{a^* \in A|$ Optimal W^T in A given $E\}$, and multi-objective decision analysis, $D^* = \{a^* \in A|G, W.E, O\}$, in which goal programming (GP), $D^* = \{a^* \in A|A \circ E = G\}$ where $\circ$ is a composite operator, is its special case. A multi-objective decision making (MODM) deals with the problem of several objectives to be optimized depicted as $D^* = \{a^* \in A|$ Optimal $W^T O$, s.t. $A = \{A \circ E = G\}\}$ (Chang [6]).

The rapid development of multi-objectives decision-making (MODM) has led to an enormous diversity of models and methods (Romero [13]). Goal programming (GP) is a one of powerful techniques for solving multi-objective optimization and has been applied to various real-life problems (Zheng et al. [17]). This model allows a decision-making take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions (Bhattacharya [2]). In practice, the DMs are interested in minimizing the non-achievement of the corresponding goal. Therefore, the GP designed to

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find a solution that minimizes the deviations between the achievement level of the objectives and the goals set for them. In the case where the goal is surpassed, the deviation will be positive and in the case of the underachievement of the goal, the deviation will be negative. A key element of GP model is the achievement function that represents a mathematical expression of the unwanted deviation variables. Nowadays, GP applications and theoretical developments have arisen (Romero [14]). The oldest and still most widely used form of achievement function for weight GP is represented as follows (Chang [5]):
(WGP)

$$
\begin{aligned}
&\text{Min} \sum_{i=1}^{n} w_i(d_i^+ + d_i^-)\\
&\text{s.t.}\ \ f_i(X) - g_i = d^+ - d_i^-,\ i = 1, 2, \ldots, n,\\
&\quad d_i^+, d_i^- \geq 0,\ i = 1, 2, \ldots, n,\\
&\quad X \in F
\end{aligned}
$$

where $f_i(X)$ is the linear function of the ith goal, $g_i(d_i^+, d_i^-)$ is linear function of the deviational variables of the aspiration level of the ith goal, F is a feasible set of constraints.

One general characteristic of all the different programs of GP models introduced so far (including weighted GP (WGP), lexicographic GP (LGP), and Ckebyshev or MINMAX GP (MGP), and extended lexicographic goal programming (EGP)) is that they have a single target value for that objective. This implies that all managerial objectives for the problem being studied can be encompassed within only a single goal (Liao [9]). However, this is not always associated with certain attributes in marketing management practice. For example, in management area many indefinite aspiration levels may exist such as “some what larger than”, “substantially lesser than”, or “around” the vague objective. In other words, in real-life problems the objectives are fuzzy (Gen et al. [7]). Fuzzy set theory provides a framework to handle the uncertainties in the objective function problems (Tsai et al. [16]). The Fuzzy GP (FGP) can be expressed as follows: Find X, so as to satisfy

$$
\begin{aligned}
&u_i(X) \gtrsim g_i \text{ or } u_i(X) \lesssim g_i,\ i = 1, 2, \ldots, I\\
&\ \text{s.t.}\ \ X \in F,
\end{aligned}
$$

where $u_i(X) \gtrsim (\lesssim) g_i$ indicates the ith fuzzy goal approximately greater than or equal to (approximately less than or equal to) the aspiration level g_i, other variables are defined as in WGP.

In the case, the FGP has the advantage of allowing for the vague aspirations of the preference concept of DMs, which can be qualified using some natural language or vague phenomena (Chang [3]). Now, in the field of fuzzy programming, the fuzzy goals characterized by their associated preference-based membership functions $f_i(X)$ for the ith fuzzy goal $f_i(X) \gtrsim g_i$ can be expressed as

$$
\pi_i(X) = \begin{cases} 1 & \text{if } f_i(X) \geq g_i,\\ \frac{f_i(X)-l_i}{g_i-l_i} & \text{if } l_i \leq f_i(X) \geq g_i,\\ 0 & \text{if } f_i(X) \leq l_i \end{cases}
$$

where l_i is the lower tolerance limit for the ith fuzzy goal. On the other hand, the membership function $f_i(X)$ for the ith fuzzy goal $f_i(X) \lesssim g_i$ can be defined as

$$
\pi_i(X) = \begin{cases} 1 & \text{if } f_i(X) \geq g_i,\\ \frac{u_i-f_i(X)}{u_i-g_i} & \text{if } g_i \leq f_i(X) \geq u_i,\\ 0 & \text{if } f_i(X) \leq u_i \end{cases}
$$

where u_i is the upper tolerance limit for the ith fuzzy goal.

For the DMs where in a fuzzy environment, the achievement of the objective goals to their aspired levels to the extent possible is actually represented by the possible achievement of their respective membership values to the highest degree (Pal et al. [12]).

In real-world there are many multi-segment selection goal problems. For example, companies/organizations often adjust their basic price to accommodate differences in products, locations, customer, and so on. Market segmentation such as museums often change a lower admission charge to senior citizens and students; wireless telecommunication business utilities vary energy rates to commercial users by time of day and weekend versus weekday (Kotler and Keller [8]; Liao [9]). This is a multi-segment selection goal problem.

Liao [9] has recently proposed a new method namely multi-segment goal programming (MSGP) for multiple decision variables coefficients problems, which allows DMs to set multi-segment aspiration levels (MSAL) selection for each goal to avoid underestimation of decision-making. The proposed idea for solving the MCDM problem with MSAL is very different from GP using membership function to manage the MCDM problem with imprecise aspiration levels of the decision variables coefficients. The conceptual expression of the MSGP achievement model can be expressed as follows:

$$\begin{aligned}
&\text{Minimize } z = \{g_1(d_1^+, d_1^-), g_2(d_2^+, d_2^-), \ldots, g_n(d_n^+, d_n^-)\}\\
&\text{Subject to, } \sum_{i=1}^{n} s_{ij} X_i + d_i^- - d_i^+ = g_i,\ j = 1, 2, \ldots, m\\
&\qquad s_{ij} = s_{i1} \text{ or } s_{i2} \text{ or} \ldots \text{or } s_{im},\ i = 1, 2, \ldots, n,\\
&\qquad X \in F\ (F \text{ is a feasible set}),
\end{aligned}$$

where s_{ij} is a decision variable coefficients, represents the multi-segment aspiration levels of jth segment of ith goal; other variables are defined as in WGP.

The MSGP can be re-expressed by the following mixed binary achievement function and it can easily be solved by commonly integer programming packages (Chang [5]; Liao [9]).
(MSGP)

$$\begin{aligned}
&\text{Min} \sum_{i=1}^{n} w_i(d_i^+ + d_i^-)\\
&\text{s.t.,} \sum_{j=1}^{m} s_{ij} B_{ij}(b) X_i + d_i^- - d_i^+ = g_i,\ i = 1, 2, \ldots, n,\\
&\quad s_{ij} B_{ij}(b) \in R_i(x),\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m,\\
&\quad d_i^+, d_i^- \geq 0,\ i = 1, 2, \ldots, n,\\
&\quad X \in F\ (F \text{ is a feasible set}),
\end{aligned} \tag{1}$$

where w_i represents the weight attached to the deviation, and s_{ij} is a decision variable coefficients, represents the multi-segment aspiration levels of jth segment of ith goal, and $B_{ij}(b)$ represents a function of binary serial number. The d_i is the deviation from the target value g_i, and $d_i^- = \max(0, g_i - s_{ij}B_{ij}(b)X_i)$ and $d_i^+ = \max(0, s_{ij}B_{ij}(b)X_i - g_i)$ are denoted under- and over- achievements of the ith goal, respectively, and $R_i(x)$ is the function of resources limitations (e.g. Chang [4]); other variables are defined as in WGP.

The remainder of this paper is organized as follows. In Section 2, for clearer descriptions of the idea, the multi-segment goal programming formulation is introduced and showed some illustrative segment examples. Section 3, the revised MSGP is proposed, and a practice application case is also provided in this section. Conclusions and points towards directions for future research are presented in Section 4.

Following the logic of the Chang [5] in the left hand side of Eq. (1) some multiplicative condition of binary variables are introduced by Liao [9] for ease deal modeling with multi-segment aspiration levels in multiple segment problems. For reduce and clear expression the binary variables number of the left hand side in (1). It is observed that to solve the MSGP model where n numbers of multi-segment aspiration levels there are $\ln(n)/\ln 2$ number of binary variables are needed. Therefore, a total of $\sum_{i=1}^{m}[\ln(s_i)/\ln 2]$ numbers of binary variables are needed (e.g. Table 1) in an equivalent model (Biswal and Acharya [1]).

In order to display the above mentioned method, this paper takes a simple example of MSGP-achievement with two segment aspiration levels as follows:

Table 1: Number of aspiration levels and binary variables (Biswal and Acharya [1])

Segment aspiration levels	Number of binary variables
$s_i = 2$	1
$s_i = 3$	$2^1 < 3 < 2^2 = 2$
$s_i = 4$	2
$s_i = 5$	$2^2 < 5 < 2^3 = 3$
$s_i = 6$	$2^2 < 6 < 2^3 = 3$
$s_i = 7$	$2^2 < 7 < 2^3 = 3$
$s_i = 8$	3

2. MSGP achievement model

The MSGP is an analytical methods devised to address decision-marking of multi-segment aspiration levels problems. In a for-profit business management, the managerial objective might well include some of the following; segment the product price, obtain distinction profits, diversify the product line, increase marketing share, and so on, in segment markets. In fact, the conflict strategies of marketing and the incompleteness of available information make if almost impossible for DMs to build a faithful mathematical model for representation of their marketing goal. The objectives are so different in organization that it really is not realistic to combine them into a single overriding objective. In order solve the problem of MSAL, the DMs attempt to set a goal to get the acceptable solutions in which DMs would interest to minimize the deviations between the achievements of goal and their aspiration levels of decision variable coefficients.

In order to detailed descriptions of the MSGP marketing application, let us consider three marketing decision problems, as depicted three figure (Figure 1, 2, and 3), respectively. There is a total product/service market M representing the global objective space of a firm can marketing and three product/service categories A, B, and C representing strategies objective spaces the firm selling in different target market (for example, has three target markets) with multiple segment aspiration levels s_{ijk} ($i = 1, 2, 3$, $j = 1, 2, 3$ and $k = 1, 2, 3$). According to the above mentioned, three strategies should be considered as follows (Liao [9]):

Strategy I: There is only one-segment aspiration level marketing strategy in each product/service categories in different target market. For example, there are three aspiration levels S_1, S_2 and S_3 corresponding to product/service categories A, B, and C (see Figure 1, modified from Liao [9]). This case is a traditional MODM problem that it can be formulated using WGP as described below:

$$
\begin{aligned}
&\text{Min } d_1^+ d_1^- + d_2^+ d_2^- + d_3^+ d_3^- \\
&\text{s. t. } s_{11}x_1 + s_{12}x_2 + s_{13}x_3 + d_1^- - d_1^+ = g_1, \\
&s_{21}x_1 + s_{22}x_2 + s_{23}x_3 + d_2^- - d_2^+ = g_2, \\
&s_{31}x_1 + s_{32}x_3 + s_{33}x_3 + d_3^- - d_3^+ = g_3, \\
&d_i^+,\ d_i^- \geq 0,\ i = 1, 2, 3, \\
&s_{ij} \in S,\ i = 1, 2, 3,\ j = 1, 2, 3, \\
&x \in F
\end{aligned}
$$

where all variables are defined as in MSGP.

Strategy II: There are two-segment aspiration levels marketing strategies in each small market. This is a case of MODM problem with an either-selection. The aspiration contribution level in segment A is to select an appropriate level from either S_{11} or S_{12}, while the aspiration contribution level in segment B is to select an appropriate level from either S_{21} or S_{22}, and in segment C is to select S_{31} or S_{32}, similarly, as depicted in Figure 2. This case cannot be solved by current GP approaches. In order to solve this marketing

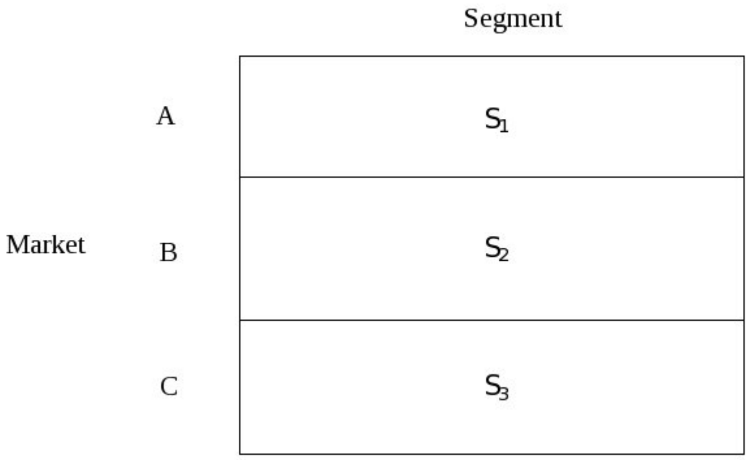

Figure 1: one-segment strategy level (Modified from Liao [9])

problem, three extra binary variables should be added as described below:

$$
\begin{aligned}
&\text{Min } w_1(d_1^+ + d_1^-) + w_2(d_2^+ + d_2^-) + w_3(d_3^+ + d_3^-)\\
&\text{s. t.}\\
&\quad (s_{11}b_1 + s_{12}(1-b_1))x_1 + s_{21}x_2 + s_{31}x_3 + d_1^- - d_2^+ = g_1,\\
&\quad s_{11}x_1 + (s_{21}b_2 + s_{22}(1-b_2))x_2 + s_{31}x_3 + d_2^- - d_2^+ = g_2,\\
&\quad s_{11}x_3 + s_{21}x_2 + (s_{31}b_3 + s_{32}(1-b_3))x_3 + d_3^- - d_3^+ = g_3,\\
&d_i^+, d_i^- \geq 0,\ i = 1,2,3,\\
&s_{ij} \in S,\ i = 1,2,3,\ j = 1,2,3,\\
&X \in F,
\end{aligned}
$$

where b_1, b_2 and b_3 are binary variables; other variables are defined as in MSGP.

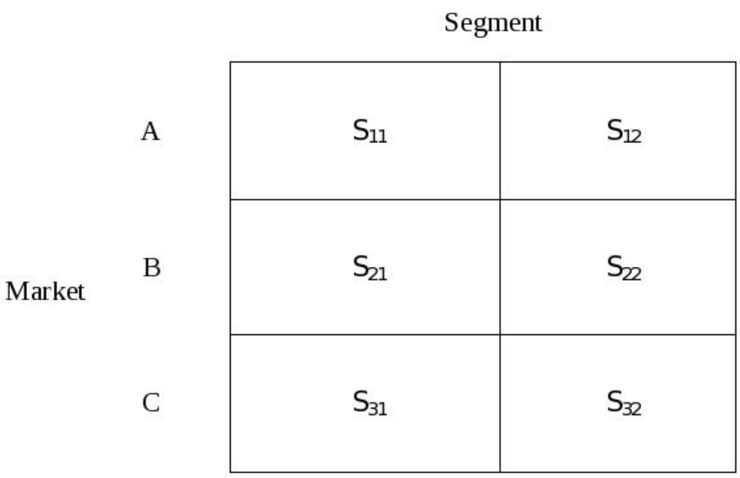

Figure 2: two-segment strategy levels (Modified from Liao [9])

Strategy III: There are multi-segment aspiration levels marketing strategies in each small market. This case is a multi-selection MODM problem. The aspiration contribution level in segment A is to select an appropriate level from either S_{11}, S_{12} or S_{13}, while the aspiration contribution level in segment B is to select an appropriate level from either S_{21}, S_{22} or S_{23}, and in segment C is to select S_{13}, S_{32} or S_{33} similarly, as depicted in Figure 3. This case cannot be solved by current GP approaches. In order to solve this marketing problem, six extra binary variables should be added as described below:

$$
\begin{aligned}
&\text{Min } w_1(d_1^+ + d_1^-) + w_2(d_2^+ d_2^-) + w_3(d_3^+ + d_3^-)\\
&\text{s. t.}\\
&(s_{11}b_1b_2 + s_{12}b_1(1-b_2) + s_{13}(1-b_1)b_2)x_1 + s_{21}x_2 + s_{31}x_3 + d_1^- - d_2^+ = g_1,\\
&s_{11}x_1 + (s_{21}b_3b_4 + s_{22}b_3(1-b_4) + s_{23}(1-b_3)b_4)x_2 + s_{31}x_3 + d_2^- - d_2^+ = g_2,\\
&s_{11}x_1 + s_{21}x_2 + (s_{31}b_5b_6 + s_{32}b_5(1-b_6) + s_{33}(1-b_5)b_6)x_3 + d_3^- - d_3^+ = g_3,\\
&d_i^+, d_i^- \geq 0,\ i = 1, 2, 3,\\
&s_{ij},\ i = 1, 2, 3,\ j = 1, 2, 3,\\
&X \in F,
\end{aligned}
$$

where b_1, b_2, b_3, b_4, b_5 and b_6 are binary variables; other variables are defined as in MSGP.

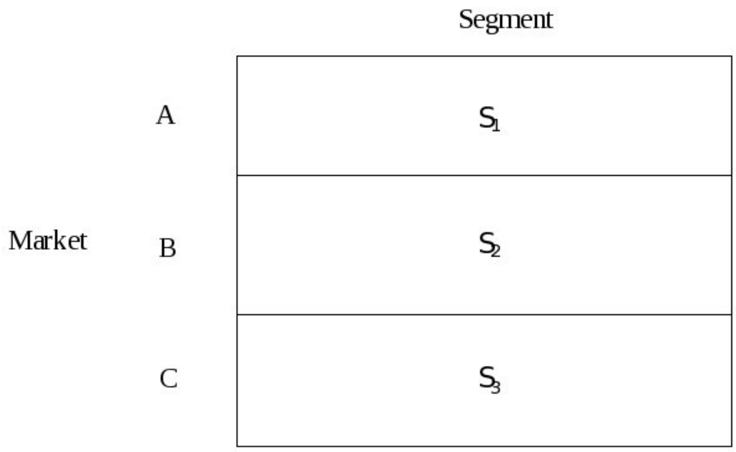

Figure 3: two-segment strategy levels (Modified from Liao [9])

The quadratic binary variables terms of b_1b_2, b_3b_4 and b_5b_6 in Strategy I–II can be linearized. For example, assume that $v = b_ib_j$, where v satisfy the following inequalities: (i) $(b_i+b_j-2)+1 \leq v \leq (2-b_i-b_j)$ (ii) $v \leq b_i$ (iii) $v \leq b_j$ and (iv) $v \geq 0$. These inequalities can be checked as: if $b_i = b_j = 1$ then $v = 1$ (from equation (i)), and equation (ii), if $b_ib_j = 0$ then $v = 0$ (from (iii)–(iv)) (Chang [4]).

Let us consider two marketing examples by Liao [9] that will be used in order to illustrate the MSGP problem with the following multi-segment aspiration levels (e.g., multiple decision variables coefficient) and constraint, which cannot be solved by current GP approaches.

Case 1: **segment example** (Liao [9]):
Goals:
(s_1) (3 or 6) $x_1 + 2x_2 + x_3 = 115$ (price segment for x_1, the more the better)
(s_2) $4x_1+$ (5 or 9) $x_2 + 2x_3 = 80$ (price segment for x_2, the more the better)
(s_3) $3.5x_1 + 5x_2+$ (7 or 10) $x_3 = 110$ (price segment for x_3, the more the better).
Constraints:
$x_2 + x_3 \geq 9$, $x_2 \geq 5$, $x_1 + x_2 + x_3 \geq 21$.

Base on the MSGP method, this problem can be formulated as the following program:

$$
\begin{aligned}
&\text{Min } d_1^+\ d_1^- + d_2^+\ d_2^- + d_3^+\ d_3^-\\
&\text{s. t.}(3b_1 + 6(1 - b_1))x_1 + 2x_2 + x_3 + d_1^+ - d_1^- = 115,\\
&\quad 4x_1 + (5b_2 + 9(1 - b_2))x_2 + 2x_3 + d_2^+ - d_2^- = 80,\\
&\quad 3.5x_1 + 5x_2 + (7b_3 + 10(1 - b_3))x_3 + d_3^+ - d_3^- = 110,\\
&\quad x_2 + x_3 \geq 9,\ x_2 \geq 5,\ x_1 + x_2 + x_3 \geq 21,\\
&\quad d_i^+, d_i^- \geq 0\ (i = 1, 2, 3)
\end{aligned}
$$

where b_1, b_2 and b_3 are binary variables; d_i^+ and d_i^- are the positive and negative deviation variables, respectively.

This problem is solved using LINGO (Schrage [15]) to obtain the optimal solutions as $(x_1, x_2, x_3, b_1, b_2, b_3) = (11.5\bar{4}, 5.0\bar{0}, 4.4\bar{6}, 0, 1, 0)$. From the results we realize that goal g_1 has $83.7\bar{0}$ achieved reached the aspiration level 115, goal g_2 has achieved reached the aspiration level 80, and goal g_3 has $109.8\bar{5}$ achieved reached the aspiration level 110.

Moreover, let us consider a MODM problem. It is slightly modified from Segment case.

Case 2: **no-segment example** (Liao [9]):

Goals:

(s_1) $3x_1 + 2x_2 + x_3 = 115$,

(s_2) $4x_1 + 9x_2 + 2x_3 = 80$,

(s_3) $3.5x_1 + 5x_2 + 7x_3 = 110$,

Constraints:

$x_2 + x_3 \geq 9$, $x_2 \geq 5$, $x_1 + x_2 + x_3 \geq 21$.

This problem is solved using LINGO (Schrage [15]) to obtain the optimal solutions as $(x_1, x_2, x_3) = (7.7\bar{1}, 5.0\bar{0}, 8.2\bar{9})$. From the results we realize that goal g_1 has a negative value $(-73.5\bar{7})$ under aspiration level 115, goal g_2 has a positive value $(+12.4\bar{3})$ over aspiration level 80, and goal g_3 has $110.0\bar{2}$ achieved reached the aspiration level 110.

It is interesting to note that the solution of Segment case is better than of No-segment caser for DMs, because the solution of Segment case is indeed balanced on the three segment goals. Therefore, the more the aspiration contribution levels the better the solutions found in the proposed MSGP approach. This forces the optimized consensus by minimizing the total deviations.

3. The revised MSGP and application

Although, the linearization techniques can solve the moderate size of multiplication terms of binary variables in the left-hand side of Eq. (3) efficiently. However, the formulation model with the multiplication terms of binary variables is difficult to implement when the problem size gets large and it is not easily understood by marketing participants. For reduce the extra binary variables used in the left-hand side of Eq. (3). Following the idea of FGP model; in this paper, a new idea of upper $(s_{ijk,\max})$ and lower $(s_{ijk,\min})$ bound of the ith aspiration level, y_{ijk} is introduced to the MSGP-achievement and y_i is the continuous variables, $s_{ijk,\min} \leq y_{ijk} \leq s_{ijk,\max}$ (Chang [3]). Therefore, the MSGP of Eq. (3) can be modified as the following two alternative types of revised MSGP-achievement functions:

Alternative MSGP-achievementType I: for the type of the more the better:

$$
\begin{aligned}
&\text{Min } \sum_{i=1}^{n} [w_i(d_i^+ + d_i^-) + \alpha(e_{ijk}^+ + e_{ijk}^-)]\\
&\text{s. t. } \sum_{i=1}^{J} s_{ijk,\max} X_j - d_i^+ + d_i^- = g_i,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, m \qquad (2)
\end{aligned}
$$

$$y_{ijk} - e^+_{ijk} e^-_{ijk} = S_{ijk,\max},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, j,\ k = 1, 2, \ldots, m, \tag{3}$$
$$s_{ijk,\min} \le y_{ijk} \le s_{ijk,\max},$$
$$d^+_i, d^-_i, e^+_{ijk}, e^-_{ijk} \ge 0, i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, j,\ k = 1, 2, \ldots, m,$$
$$X \in F,$$

where d^+_i and d^-_i are the positive and negative deviation attached to the ith goal

$$|\sum_{j=1}^{J} s_{ijk,\max} X_j - g_i|$$

in Eq. (2); e^+_{ijk} and e^-_{ijk} are the positive and negative deviation attached to $|y_{ijk} - s_{ijk,\max}|$ in Eq. (3); α_i is the weight attached to the sum of the deviation of $|y_{ijk} - s_{ijk,\max}|$; other variables are defined as in MSGP (Liao and Kao [11]; Liao [10]).

Alternative MSGP-achievementType II: for the type of the less the better:

$$\text{Min } \sum_{i=1}^{n} [w_i(d^+_i + d^-_i) + \alpha(e^+_{ijk} + e^-_{ijk})]$$
$$\text{s. t. } \sum_{j=1}^{J} s_{ijk,\min} X_j - d^+_i + d^-_i = g_i,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, m \tag{4}$$
$$y_{ijk} - e^+_{ijk} + e^-_{ijk} = S_{ijk,\min},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, j,\ k = 1, 2, \ldots, m, \tag{5}$$
$$s_{ijk,\min} \le y_{ijk} \le s_{ijk,\max},$$
$$d^+_i, d^-_i, e^+_{ijk}, e^-_{ijk} \ge 0,\ i = 1, 2, \ldots, J,\ j = 1, 2, \ldots, j,\ k = 1, 2, \ldots, m,$$
$$X \in F,$$

where d_i+ and d^-_i are the positive and negative deviation attached ith goal $\sum_{j=1}^{J} S_{ijk,\min} X_j - gi$ in Eq. (4); e^+_{ijk} and e^-_{ijk} are the positive and negative deviation attached $|y_{ijk} - S_{ijk,\min}|$ to in Eq. (5); α_i is the weight attached to the sum of the deviation of $|y_{ijk} - S_{ijk,\min}|$; other variables are defined as in MSGP (Liao and Kao [11]; Liao [10]).

In order to simplicity but without loss of generality and following the idea of proof process by Chang [5], let us take alternative MSGP-achievementtype I for instance to prove that the MSGP-achievement and alternative MSGP-achievement are equivalent as follows.

Proposition 3.1. *Alternative MSGP achievement type I and MSGP achievement with maximization of $\sum_{j=1}^{J} s_{ijk} X_j$ are equivalent in the sense that have same optimal solutions.*

Proof. (i) y_{ijk} in Eq. (3) will be forced to approach $s_{ijk,\max}$ as close as possible because e^+_{ijk} and e^-_{ijk} should be minimized in the objective function of the minimization problem.
(ii) $\sum_{j=1}^{J} s_{ijk,\max} X_j$ in Eq. (2) will be forced to approach g_i as close as possible because d^+_i and d^-_i should be minimized in the objective function of the minimization problem.

According to Proof (i) and (ii), will be forced to approach g_i as close as possible. The purpose of alternative MSGP-achievement—type I is to minimize the deviations (i.e., d^+_i and d^-_i) between $\sum_{j=1}^{J} s_{ijk,\max} X_j$ and g_i (i.e., the achievement of ith goal and their aspiration segment level, y_{ijk}) where y_{ijk} is a continuous variable, $s_{ijk,\min} \le y_{ijk} \le s_{ijk,\max}$. In addition, the idea of upper $s_{ijk,\max}$ bound of the ith aspiration segment level, y_{ijk}, is to minimize the deviation (i.e., e^+_{ijk} and r^-_{ijk}) between the s_{ijk} and y_{ijk}. It will be obvious that the behavior of alternative MSGP-achievement—type is equivalent to the MSGP-achievement with maximization of $\sum_{j=1}^{J} s_{ijk} X_j$, □

Now, let us illustrate a marketing case for a revised MSGP problem application, which can not be solved by current GP techniques as follows:

ABC company is manufacturing and launching three products x_1, x_2 and x_3 to three new markets with three goals. The CEO (chief executive officer) has established the goals and marketing strategies of (G_1) achieving the sales amount at $110 million dollars from these products sales in first market and the price segment as little as $3 or as much as $6 (or price interval [3, 6]) per unit of product x_1 under two-segment aspiration levels marketing strategies;

(G_2) achieving the sales amount at $85 million dollars from these products sales in second market and the price at $8.5 per unit of product x_2 under one-segment aspiration level marketing strategy;

(G_3) achieving the sales amount at it $130 million dollars from these products sales in third market and the price segment as little as $7 or as much as $11 (or price interval [7, 11]]) per unit of product x_3, and the price segment as little as $3.5 or as much as $6.5 (or price interval [3.5, 6.5]) per unit of product under multi-segment aspiration levels marketing strategies;

(G_4) maintaining the current total capacity of 35 salespeople with products x_1, x_2 and x_3 taking 5, 3 and 4 sales people, respectively;

(G_5) holding the budget of marketing cost to less than 55 million with products, x_1 and x_2 taking 7 and 5 million, respectively. Permitting the budget of marketing cost at new product x_3 as little as $5.5 or as much as $8 (or budget interval [5.5, 8]).

In addition, the selling price for products x_1, x_2 and x_3 under the strategy of nosegment marketing are $4.5, $8.5 and $9, respectively. For this problem, the related functions and parameters are listed below:
Goals:
$f_1(x) = [3, 5]x_1 + 8.5x_2 + 9x_3 = 110$
(first market sales amount, the more the better)
$f_1(x) = 4.5x_1 + 8.5x_2 + 9x_3 = 85$
(second market sales amount, the more the better)
$f_3(x) = [3.5, 6.5]x_1 + 8.5x_2 + [7, 10]x_3 = 130$
(third market sales amount, the more the better)
$f_4(x) = 5x_1 + 3x_2 + 4x_3 = 35$ (salespeople goal)
$f_5(x) = 7x_1 + 5x_2 + [5.5, 8]x_3 = 55$
(marketing cost budget, the less the better)

Base on the revised MSGP method, this problem can be formulated as the following program:

$$
\begin{aligned}
&\text{Min} \sum_{i=1}^{5}(d_i^+ + d_i^-) + \sum_{j=1}^{4}(e_i^+ + e_i^-) & \\
&\text{s.t.} y_1x_1 + 8.5x_2 + 9x_3 - d_1^+ + d_1^- = 110 && \text{for goal } G_1 \\
&\quad y_1 - e_1^+ e_1^- = 5 && \text{for } |y_i - s_{ijk,\max}| \\
&\quad 3 \le y_1 \le 5 && \text{for bound of } y_1 \\
&\quad 4.5x_1 + 8.5x2 + 9x_3 - d_2^+ + d_2^- = 85 && \text{for goal } G_2 \\
&\quad y_2x_1 + 8.5x_2 + y_3x_3 - d_3^+ + d_3^- = 130 && \text{for goal } G_3 \\
&\quad y_2 = e_2^+ + e_2^- = 6.5 && \text{for } |y_i - s_{ijk,\max}| \\
&\quad 3.5 \le y_2 \le 6.5 && \text{for bound of } y_2 \\
&\quad y_3 - e_3^+ + e_3^- = 10 && \\
&\quad 7 \le y_3 \le 10 && \text{for } |y_i - s_{ijk,\max}| \\
&\quad 5x_1 + 3x_2 + 4x_3 - d_4^+ + d_4^- = 35 && \text{for bound of } y_3 \\
&\quad 7x_1 + 5x_2 + 8x_3 - d_5^+ + d_5^- = 55 && \text{for sales people goal} \\
&\quad y_4 - e_4^+ + e_4^- = 5.5 && \text{for cost budget} \\
&\quad 5.5 \le y_4 \le 8 && \text{for } |y_i - s_{ijk,\min}| \\
&\quad d_i^+, d_i^- \ge 0,\ (i = 1, 2, 3, 4, 5) && \text{for bound of } y_4 \\
&\quad e_j^+, e_j^- \ge 0,\ (j = 1, 2, 3, 4) &&
\end{aligned}
$$

This problem is solved using LINGO (Schrage [15]) on a Pentium (R) 4 CPU 2.00 GHz-based microcomputer in a few seconds of computer time. The optimal solutions are as $(x_1, x_2, x_3, y_1, y_2, y_3, y_4) = (0, 7.8\bar{6}, 2.8\bar{6}, 6.5, 11, 5.5)$ and

$$(f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)) = 92.5\bar{0}, 92.5\bar{0}, 95.35, 35, 55).$$

That is, the price segment strategies are fully satisfied in three new markets with three goals. On the other hand; from the results we realize that goal $G_1(92.5)$, $G_2(92.5)$ and $G_3(93.35)$ has achieved the aspiration level 110, 85 and 130, respectively.

From the marketing strategies of ABC company, that market-penetration strategy and market-development strategy would be reach; therefore, the salespersons' goal and marketing cost budget are more important then other goals. This mean that the salespersons' goal and marketing cost budget goal the lesser the better should be achieved and the other goals are not. Therefore, the salespersons' goal (G_4) and marketing cost budget goal (G_5) are fully satisfied and reached the aspiration levels are 35 and 55.

4. Conclusions

Marketing decision making such as time segment, price discrimination, location and channel segment, customer segment designs are often formulated as multi-segment aspiration level problems. This is a case of multi-segment MODM problem. To the best knowledge of our, this problem cannot be solved by current GP approaches. In this paper, a new MSGP model is proposed to deal with the multiple aspiration levels marketing segment problems, which could certainly obtain a solution close to the DMs multi-segment aspiration levels. In addition, for improve the involvement multiplicative terms of binary variables in MSGP model, a revised MSGP method and application case has introduced. This modified it easier to implement using common linear programming packages and easier to understand by marketing DMs application. As a result, the practical utility of current GP approach has been expanded in this paper. The promising results stimulate the need for future research on nonlinear function of the aspiration levels or more practice application in real-world.

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