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Estimation of reliability in multicomponent stress-strength model based on Rayleigh distribution

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Abstract. A multicomponent system of k components having strengths following k-independently and identically distributed random variables and each component experiencing a random stress Y is considered. The system is regarded as alive only if at least s out of k $(s < k)$ strengths exceed the stress. The reliability of such a system is obtained when strength, stress variates are given by Rayleigh distribution with different scale parameters. The reliability is estimated using the Moment method and ML method of estimation when samples drawn from strength and stress distributions. The reliability estimators are compared asymptotically. The small sample comparison of the reliability estimates is made through Monte Carlo simulation. Using real data sets we illustrate the procedure.

1. Introduction

The Rayleigh distribution is a special case of the two parameter Weibull distribution and a suitable model for life-testing studies. The Rayleigh distribution has the most commonly used distribution in reliability and life testing (see Lawless (2003)). This distribution has several desirable properties and nice physical interpretations and it has increasing failure rate. The various applications of this distribution were studied by Polovko (1968), Gross and Clark (1975), Lee et al. (1980) and Siddiqui (1962). Having found a considerable number of research articles about Rayleigh distribution applications in reliability published in various periodicals of wide reputation, we motivated to study the multivariate stress-strength reliability estimation based on this distribution. The Rayleigh distribution has the following density function

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}},\ x > 0, \sigma > 0,$$

and the distribution function

$$F(x;\sigma) = 1 - e^{\frac{-x^2}{2\sigma^2}},\ x > 0, \sigma > 0,$$

where X is a continuous random variable defined over $(0,\infty)$ and σ is the scale parameter. The purpose of this paper is to study the reliability in a multicomponent stress-strength based on X, Y being two independent random variables, where X and Y follows Rayleigh distributions with parameters σ_1 and σ_2 respectively.

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A multicomponent system with k components has strengths following $k-$independently and identically distributed random variables $X_1, X_2, \ldots, X_k$ and each component experiences a random stress Y. The system is regarded as alive only if at least k $(s < k)$ strengths exceed the stress. Let the random samples $Y, X_1, X_2, \ldots, X_k$ be independent, $G(y)$ be the continuous distribution function of Y and $F(x)$ be the common continuous distribution function of $X_1, X_2, \ldots, X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is given by

$$R_{s,k} = P[\text{at least } s \text{ of the } X_1, X_2, \ldots, X_k \text{ exceed } Y] = \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} [1 - G(y)]^i \; [G(y)]^{(k-i)} \; dF(y), \quad (1)$$

where $X_1, X_2, \ldots, X_k$ are identically independently distributed (i.i.d.) with common distribution function $F(x)$ and subjected to the common random stress Y. The probability in (1) is called reliability in a multicomponent stress-strength model (Bhattacharyya and Johnson (1974)). The reliability estimation of a single component stress-strength version has been considered by several authors assuming various lifetime distributions for the stress-strength random variates. Enis and Geisser (1971), Downtown (1973), Awad and Gharraf (1986), McCool (1991), Nandi and Aich (1994), Surles and Padgett (1998), Raqab and Kundu (2005), Kundu and Gupta (2005, 2006), Raqab et al. (2008), Kundu and Raqab (2009). The reliability in a multicomponent stress-strength was developed by Bhattacharyya and Johnson (1974), Pandey and Borhan (1985) and the references therein cover the study of estimating $P(Y < X)$ in many standard distributions assigned to one or both of stress and strength variates. Recently Rao and Kantam (2010) studied estimation of reliability in multicomponent stress-strength for the log-logistic distribution and Rao (2012) developed an estimation of reliability in multicomponent stress-strength based on generalized exponential distribution.

Suppose a system, with k identical components, functions if at least s $(1 \leq s \leq k)$ or more of the components simultaneously operate. In its operating environment, the system is subjected to a stress Y which is a random variable with distribution function $G(\cdot)$. The strengths of the components, that is the minimum stresses to cause failure, are independent and identically distributed random variables with distribution function $F(\cdot)$. Then the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (1). The estimation of survival probability in a multicomponent stress-strength system when the stress and strength variates are following Rayleigh distribution is not paid much attention. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to Rayleigh distribution. The expression for and maximum likelihood (ML) estimates and Method of Moments (MOM) estimates of the parameters are provided in Section 2. The MOM and MLE are employed to obtain the asymptotic distribution and confidence intervals for $R_{s,k}$. The small sample comparisons made through Monte Carlo simulations in Section 3. Also, using real data, we illustrate the estimation process. Finally, the conclusion and comments are provided in Section 4.

2. Different methods of estimation of parameters in $R_{s,k}$

Let X, Y be two independent random variables follows Rayleigh distributions with parameters σ_1 and σ_2 respectively. The reliability in multicomponent stress-strength for Rayleigh distribution using (1), we get

$$\begin{aligned}
R_{s,k} &= \sum_{i=s}^{k} \binom{k}{i} \int_0^{\infty} \left[e^{\frac{-y^2}{2\sigma_2^2}}\right]^i \left[1 - e^{\frac{-y^2}{2\sigma_2^2}}\right]^{k-i} \frac{y}{\sigma_1^2} e^{\frac{-y^2}{2\sigma_1^2}} dy \\
&= \sum_{i=s}^{k} \binom{k}{i} \int_0^{1} [t^{\lambda^2}]^i \; [1 - t^{\lambda^2}]^{k-i} dt, \qquad \text{where} \quad t = e^{\frac{-y^2}{2\sigma_1^2}} \quad \text{and} \quad \lambda = \frac{\sigma_1^2}{\sigma_2^2} \\
&= \frac{1}{\lambda^2} \sum_{i=s}^{k} \binom{k}{i} \int_0^{1} [1 - z]^{k-i} \; [z]^{(1+\frac{1}{\lambda^2}-1)} dz, \qquad \text{if} \quad z = t^{\lambda^2} \\
&= \frac{1}{\lambda^2} \sum_{i=s}^{k} \binom{k}{i} B\left(k - i + 1, i + \frac{1}{\lambda^2}\right).
\end{aligned}$$

After the simplification we get

$$R_{s,k} = \frac{1}{\lambda^2}\sum_{i=s}^{k}\frac{k!}{i!}\left[\prod_{j=1}^{k}(\frac{1}{\lambda^2}-j)\right]^{-1}, \quad \text{since } k \text{ and } i \text{ are integers.} \tag{2}$$

The probability in (2) is called reliability in a multicomponent stress-strength model. If σ_1 and σ_2 are not known, it is necessary to estimate σ_1 and σ_2 to estimate $R_{s,k}$. In this paper we estimate σ_1 and σ_2 by moment method and ML method. The estimates are substituted in λ to get an estimate of $R_{s,k}$ using equation (2). The theory of methods of estimation is explained below.

It is well known that the method of Maximum Likelihood Estimation (MLE) has invariance property. When the method of estimation of parameter is changed from ML to any other traditional method, this invariance principle does not hold good to estimate the parametric function. However, such an adoption of invariance property for other optimal estimators of the parameters to estimate a parametric function is attempted in different situations by different authors. Travadi and Ratani (1990), Kantam and Rao (2002) and the references therein are a few such instances. In this direction, we have proposed some estimators for the reliability of multicomponent stress-strength model by considering the estimators of the parameters of stress and strength distributions by ML method and Moment method of estimation in Rayleigh distribution.

2.1. Method of maximum likelihood estimation (MLE)

Let $X_1, X_2, \ldots, X_n$; $Y_1, Y_2, \ldots, Y_m$ be two ordered random samples of size n, m respectively on strength and stress variates each following Rayleigh distribution with scale parameters σ_1 and σ_2. The log-likelihood function of the observed sample is

$$L(\sigma_1,\sigma_2) = -2n\ln\sigma_1 - 2m\ln\sigma_2 - \frac{1}{2\sigma_1^2}\sum_{i=1}^{n}x_i - \frac{1}{2\sigma_2^2}\sum_{j=1}^{m}y_j + \sum_{i=1}^{n}\ln x_i + \sum_{j=1}^{m}\ln y_j.$$

The MLEs $\hat{\sigma_1}$ and $\hat{\sigma_2}$ of σ_1 and σ_2 respectively can be obtained as

$$\frac{\partial L}{\partial \sigma_1} = 0 \Rightarrow \hat{\sigma_1} = \sqrt{\frac{1}{2n}\sum_{i=1}^{n} x_i^2},$$

$$\frac{\partial L}{\partial \sigma_2} = 0 \Rightarrow \hat{\sigma_2} = \sqrt{\frac{1}{2m}\sum_{i=1}^{m} y_i^2}.$$

Once we obtain $\hat{\sigma_1}$ and $\hat{\sigma_2}$ the ML estimator of $R_{s,k}$ becomes $\hat{R}_{s,k}$ with λ is replaced by $\hat{\lambda}$ in (2).

2.2. Method of moment estimation (MOM)

We know that, if $\overline{x}$ and $\overline{y}$ are the sample means of samples on strength and stress variates then moment estimators of σ_1 and σ_2 are $\tilde{\sigma_1} = \overline{x}\sqrt{\frac{2}{\Pi}}$ and $\tilde{\sigma_2} = \overline{y}\sqrt{\frac{2}{\Pi}}$, respectively. The MOM estimator of $R_{s,k}$, we propose here is $\tilde{R}_{s,k}$ with λ is replaced by $\tilde{\lambda} = \frac{\tilde{\sigma_1}}{\tilde{\sigma_2}}$ in (2).

To obtain the asymptotic confidence interval for $R_{s,k}$, we proceed as follows. The asymptotic variance of the MLEs are given by

$$V(\hat{\sigma_1}) = \left[E\left(-\frac{\partial^2 L}{\partial\sigma_1^2}\right)\right]^{-1} = \frac{\sigma_1^2}{4n} \quad \text{and} \quad V(\hat{\sigma_2}) = \left[E\left(-\frac{\partial^2 L}{\partial\sigma_2^2}\right)\right]^{-1} = \frac{\sigma_2^2}{4m}.$$

Under central limit property for i.i.d. variates, the asymptotic distribution of the moment estimators are normal with the asymptotic variances are given by

$$V(\tilde{\sigma_1}) = V\left(\frac{\overline{x}}{\sqrt{\frac{\Pi}{2}}}\right) = \left(\frac{4-\Pi}{\Pi}\right)\frac{\sigma_1^2}{n} \quad \text{and} \quad V(\tilde{\sigma_2}) = V\left(\frac{\overline{y}}{\sqrt{\frac{\Pi}{2}}}\right) = \left(\frac{4-\Pi}{\Pi}\right)\frac{\sigma_2^2}{m}.$$

The asymptotic variance (AV) of an estimate of $R_{s,k}$ which a function of two independent statistics (say) $\hat{\sigma_1}$ and $\hat{\sigma_2}$ is given by Rao (1973) as

$$AV(\hat{R}_{s,k}) = V(\hat{\sigma_1})\left[\frac{\partial R_{s,k}}{\partial \sigma_1}\right]^2 + V(\hat{\sigma_2})\left[\frac{\partial R_{s,k}}{\partial \sigma_2}\right]^2, \tag{3}$$

where t_1 and t_2 are to be taken in two different ways namely, ML and MOM estimators of σ_1 and σ_2 respectively. Thus from (3), the asymptotic variance of $\hat{R}_{s,k}$ can be obtained by replacing t_1 and t_2 with ML estimators of σ_1 and σ_2 whereas asymptotic variance of $\tilde{R}_{s,k}$ can be obtained by replacing t_1 and t_2 with MOM estimators of σ_1 and σ_2.

To avoid the difficulty of derivation of $R_{s,k}$, we obtain the derivatives of $R_{s,k}$ for (s,k)=(1,3) and (2,4) separately, they are given by

$$\frac{\partial R_{1,3}}{\partial \sigma_1} = \frac{-12\hat{\lambda}^5\left(11\hat{\lambda}^4+12\hat{\lambda}^2+3\right)}{\sigma_2\left[\left(\hat{\lambda}^2+1\right)\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\right]^2} \quad \text{and} \quad \frac{\partial R_{1,3}}{\partial \sigma_2} = \frac{12\hat{\lambda}^6\left(11\hat{\lambda}^4+12\hat{\lambda}^2+3\right)}{\sigma_2\left[\left(\hat{\lambda}^2+1\right)\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\right]^2}$$

$$\frac{\partial R_{2,4}}{\partial \sigma_1} = \frac{-48\hat{\lambda}^5\left(26\hat{\lambda}^4+18\hat{\lambda}^2+3\right)}{\sigma_2\left[\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\left(4\hat{\lambda}^2+1\right)\right]^2} \quad \text{and} \quad \frac{\partial R_{2,4}}{\partial \sigma_2} = \frac{48\hat{\lambda}^6\left(26\hat{\lambda}^4+18\hat{\lambda}^2+3\right)}{\sigma_2\left[\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\left(4\hat{\lambda}^2+1\right)\right]^2}.$$

Thus

$$AV\left(\hat{R}_{1,3}\right) = \frac{36\hat{\lambda}^{12}\left(11\hat{\lambda}^4+12\hat{\lambda}^2+3\right)^2}{\left[\left(\hat{\lambda}^2+1\right)\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\right]^4}\left(\frac{1}{n}+\frac{1}{m}\right),$$

$$AV\left(\tilde{R}_{1,3}\right) = \frac{144\tilde{\lambda}^{12}\left(11\tilde{\lambda}^4+12\tilde{\lambda}^2+3\right)^2}{\left[\left(\tilde{\lambda}^2+1\right)\left(2\tilde{\lambda}^2+1\right)\left(3\tilde{\lambda}^2+1\right)\right]^4}\left(\frac{4-\Pi}{\Pi}\right)\left(\frac{1}{n}+\frac{1}{m}\right),$$

$$AV\left(\hat{R}_{2,4}\right) = \frac{576\hat{\lambda}^{12}\left(26\hat{\lambda}^4+18\hat{\lambda}^2+3\right)^2}{\left[\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\left(4\hat{\lambda}^2+1\right)\right]^4}\left(\frac{1}{n}+\frac{1}{m}\right)$$

and

$$AV\left(\tilde{R}_{2,4}\right) = \frac{2304\tilde{\lambda}^{12}\left(26\tilde{\lambda}^4+18\tilde{\lambda}^2+3\right)^2}{\left[\left(2\tilde{\lambda}^2+1\right)\left(3\tilde{\lambda}^2+1\right)\left(4\tilde{\lambda}^2+1\right)\right]^4}\left(\frac{4-\Pi}{\Pi}\right)\left(\frac{1}{n}+\frac{1}{m}\right).$$

As $n \to \infty$, $m \to \infty$, we have that $\frac{\hat{R}_{s,k}-R_{s,k}}{AV(\hat{R_{s,k}})} \xrightarrow{d} N(0,1)$ and the asymptotic confidence 95% confidence interval for $R_{s,k}$ is given by

$$\hat{R}_{s,k} \pm 1.96\sqrt{AV(\hat{R}_{s,k})}.$$

The asymptotic confidence 95% confidence interval for $R_{1,3}$ using ML and MOM estimators are respectively given by

$$\hat{R}_{1,3} \pm 1.96\frac{6\hat{\lambda}^6\left(11\hat{\lambda}^4+12\hat{\lambda}^2+3\right)}{\left[\left(\hat{\lambda}^2+1\right)\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)},$$

$$\tilde{R}_{1,3} \pm 1.96 \frac{12\tilde{\lambda}^6\left(11\tilde{\lambda}^4+12\tilde{\lambda}^2+3\right)}{\left[\left(\tilde{\lambda}^2+1\right)\left(2\tilde{\lambda}^2+1\right)\left(3\tilde{\lambda}^2+1\right)\right]^2}\sqrt{\left(\frac{4-\Pi}{\Pi}\right)}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}.$$

The asymptotic confidence 95% confidence interval for $R_{2,4}$ using ML and MOM estimators are respectively given by

$$\hat{R}_{2,4} \pm 1.96 \frac{24\hat{\lambda}^6\left(26\hat{\lambda}^4+18\hat{\lambda}^2+3\right)}{\left[\left(2\hat{\lambda}^2+1\right)\left(3\hat{\lambda}^2+1\right)\left(4\hat{\lambda}^2+1\right)\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)},$$

$$\tilde{R}_{2,4} \pm 1.96 \frac{48\tilde{\lambda}^6\left(26\tilde{\lambda}^4+18\tilde{\lambda}^2+3\right)}{\left[\left(2\tilde{\lambda}^2+1\right)\left(3\tilde{\lambda}^2+1\right)\left(4\tilde{\lambda}^2+1\right)\right]^2}\sqrt{\left(\frac{4-\Pi}{\Pi}\right)}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}.$$

3. Simulation study and data analysis

3.1. Simulation study

In this subsection we present some results based on Monte Carlo simulations to compare the performance of the $R_{s,k}$ using for different sample sizes. 3000 random sample of size 10(5)35 each from stress and strength populations are generated for (σ_1, σ_2)=(3.0, 1.0), (2.5, 1.0), (2.0, 1.0), (1.5, 1.0), (1.0, 1.0), (1.5, 2.0), (1.5, 2.5) and (1.5, 3.0) as proposed by Bhattacharyya and Johnson (1974). The ML estimators and MOM estimators of σ_1 and σ_2 are then substituted in λ to get the reliability in a multicomponent reliability for $(s, k) = (1, 3), (2, 4)$. The average bias and average mean square error (MSE) of the reliability estimates over the 3000 replications for two methods of estimation are given in Tables 1 and 2. Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ for two methods of estimation are given in Tables 3 and 4. The true values of reliability in multicomponent stress-strength with the given combinations of (σ_1, σ_2) for $(s, k) = (1, 3)$ are 0.178, 0.242, 0.344, 0.507, 0.750, 0.917, 0.971, 0.989, 0.995 and for $(s, k) = (2, 4)$ are 0.111, 0.155, 0.228, 0.359, 0.600, 0.828, 0.929, 0.969, 0.986 respectively. Thus the true value of reliability in multicomponent stress-strength increases as σ_2 increases for a fixed σ_1 whereas reliability in multicomponent stress-strength decreases as σ_1 increases for a fixed σ_2 in both the cases of (s, k). Therefore, the true value of reliability is increases as λ decreases and vice versa. The average bias and average MSE are decreases as sample size increases for both methods of estimation in reliability. It verifies the consistency property of the MLE of $R_{s,k}$. The absolute bias of MLE shows less than the absolute bias of moment estimator in most of the parametric and sample combinations. Also the bias is negative when $\sigma_1 \leq \sigma_2$ and other cases bias is positive in both situations of (s, k). With respect to MSE also MLE shows first preference than moment method of estimation. Whereas, among the parameters the absolute bias and MSE are increases as σ_1 increases for a fixed value of σ_2 in both the cases of (s, k) and the absolute bias and MSE are decreases as σ_2 increases for a fixed value of σ_1 in both the cases of (s, k). The average length of the confidence interval is also decreases as the sample size increases. The average length of the confidence interval based on MLE shows shortest average length than by using moment method of estimation. The simulated actual coverage probability is close to the nominal value in all cases but slightly less than 0.95 in most of the combinations for both methods of estimation. Overall, the performance of the confidence interval is quite good for all combinations of parameters. Whereas, among the parameters we observed the same phenomenon for average length and average simulated actual coverage probability that we observed in case of average bias and MSE. The simulation results also show that there is no considerable difference in the average bias and average MSE for different choices of the parameters. The same phenomenon is observed for the average lengths and coverage probabilities of the confidence intervals.

3.2. Data analysis

In this sub section we analyze two real data sets and demonstrate how the proposed methods can be used in practice. The first data set reported by Wang (2000) and second data set given by Lawless (2003). We fit the Rayleigh distribution to the two data sets separately. The first data set (Wang (2000)) presented here arose in failure time of 18 devices and they are as follows: Data set I: 5, 11, 21, 31, 46, 75, 98,122, 145,165, 195, 224, 245, 293, 321, 330, 350, 420. Wang (2000) has fitted this data to Burr XII distribution. The second data set is obtained from Lawless (2003) and it represents the number of revolution before failure of each of 23 ball bearings in the life tests and they are as follows: Data Set II: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40. Gupta and Kundu (2001a) studied the validity of this data for gamma, Weibull and generalized exponential distributions.

Before analyzing further, we checked the validity of the models. We used the Kolmogorov-Smirnov (K–S) tests for each data set to fit the Rayleigh distribution. It is observed that for first data set the K–S distance between the empirical distribution function and the fitted distribution functions is 0.22319 with the corresponding p– value 0.28647 and for the second data set the K–S distance between the empirical distribution function and the fitted distribution functions is 0.14165 with the corresponding p–value 0.69348. It indicates that the Rayleigh model fits quite well to both the data sets. We plot the empirical survival functions and the fitted survival functions in Figures 1 and 2 for data set I and data set II respectively.

The ML and MOM estimates for real data sets are $\hat{\sigma_1} = 137.8$, $\hat{\sigma_2} = 57.624$ and $\tilde{\sigma_1} = 137.25$, $\tilde{\sigma_2} = 57.61$, respectively. Basing on ML estimates of σ_1 and σ_2 the MLE of $R_{s,k}$ become $\hat{R}_{1,3}$=0.261981 and $\hat{R}_{2,4}$=0.16857. The 95% confidence intervals in this case for $R_{1,3}$ become (0.131533, 0.392429) and for $R_{2,4}$ become (0.076964, 0.260176). Similarly, Base on moment estimates of σ_1 and σ_2 the MOM of $R_{s,k}$ become $\tilde{R}_{1,3} = 0.26198$ and $\tilde{R}_{1,3} = 0.168569$. The 95% confidence intervals for $R_{1,3}$ become (0.125888, 0.398072) and for $R_{2,4}$ become (0.07289, 0.264248). From the real life data, we conclude that one out of three component system reliability is more than the two out of four component system reliability for both methods of estimation. The length of the confidence interval is also more for one out of three component system reliability than the two out of four component system reliability.

4. Conclusions

In this paper, we have studied the multicomponent stress-strength reliability for Rayleigh distribution when both of stress and strength variates follow the same population. Also, we have estimated asymptotic confidence interval for multicomponent stress-strength reliability using MLE and moment method of estimation. The simulation results indicates that the average bias and average MSE are decreases as sample size increases for both methods of estimation in reliability. Among the parameters the absolute bias and MSE are increases (decreases) as σ_1 increases (σ_2 increases) in both the cases of (s, k). The length of the confidence interval is also decreases as the sample size increases and coverage probability is close to the nominal value in all sets of parameters considered here. Using real data, we illustrated the estimation process.

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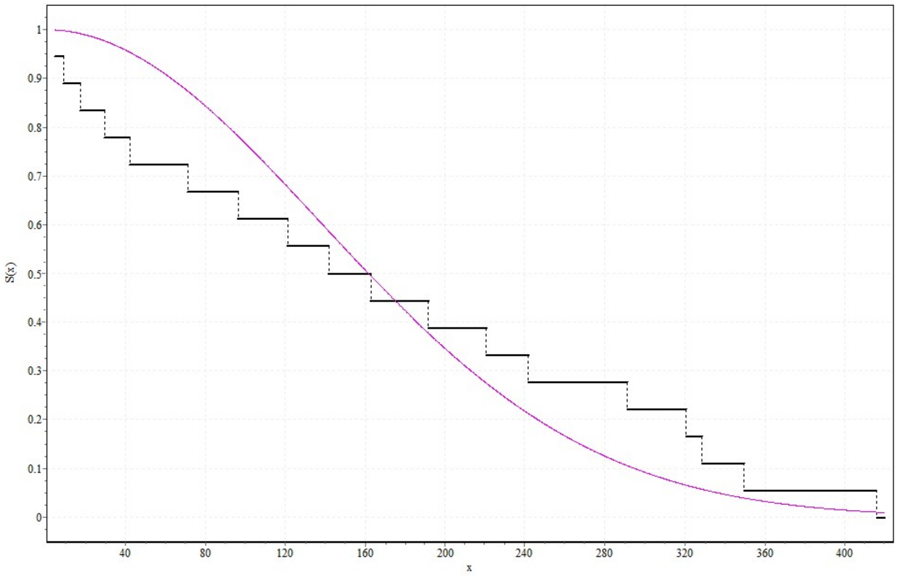

Figure 1: The empirical and fitted survival functions for the Data Set I

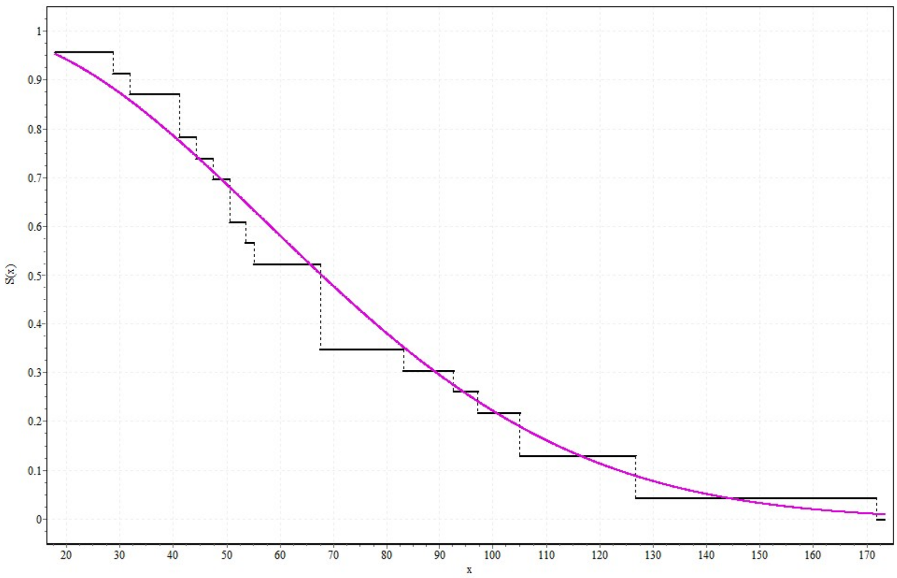

Figure 2: The empirical and fitted survival functions for the Data Set II

Table 1: Average bias of the simulated estimates of $R_{s,k}$

		(σ_1,σ_2)						
(s,k)	(n,m)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)
	(5,5)	0.02544	0.02161	0.00610	-0.02473	-0.03333	-0.02342	-0.01421
		0.02410	0.02072	0.00629	-0.02292	-0.03122	-0.02187	-0.01319
	(10,10)	0.01407	0.01290	0.00538	-0.01152	-0.01627	-0.01072	-0.00595
		0.01334	0.01229	0.00524	-0.01071	-0.01520	-0.00997	-0.00550
	(15,15)	0.00867	0.00753	0.00180	-0.01034	-0.01283	-0.00811	-0.00438
		0.00821	0.00726	0.00210	-0.00903	-0.01143	-0.00721	-0.00388
(1,3)	(20,20)	0.00728	0.00677	0.00284	-0.00641	-0.00891	-0.00570	-0.00307
		0.00586	0.00520	0.00144	-0.00693	-0.00873	-0.00548	-0.00292
	(25,25)	0.00739	0.00735	0.00444	-0.00343	-0.00629	-0.00415	-0.00224
		0.00665	0.00659	0.00390	-0.00329	-0.00582	-0.00381	-0.00205
	(30,30)	0.00434	0.00388	0.00113	-0.00499	-0.00626	-0.00387	-0.00203
		0.00381	0.00334	0.00076	-0.00490	-0.00595	-0.00365	-0.00191
	(35,35)	0.00314	0.00259	0.00009	-0.00512	-0.00581	-0.00350	-0.00181
		0.00302	0.00257	0.00034	-0.00443	-0.00517	-0.00313	-0.00162
	(5,5)	0.02357	0.02590	0.02109	-0.00559	-0.03279	-0.03324	-0.02487
		0.02212	0.02450	0.02028	-0.00471	-0.03062	-0.03114	-0.02324
	(10,10)	0.01226	0.01417	0.01278	-0.00081	-0.01608	-0.01620	-0.01147
		0.01159	0.01343	0.01218	-0.00059	-0.01502	-0.01512	-0.01067
	(15,15)	0.00773	0.00879	0.00739	-0.00276	-0.01339	-0.01267	-0.00869
		0.00726	0.00831	0.00715	-0.00206	-0.01187	-0.01130	-0.00773
(2,4)	(20,20)	0.00624	0.00731	0.00672	-0.00051	-0.00893	-0.00885	-0.00612
		0.00513	0.00592	0.00513	-0.00165	-0.00908	-0.00863	-0.00588
	(25,25)	0.00611	0.00736	0.00738	0.00172	-0.00577	-0.00631	-0.00445
		0.00551	0.00663	0.00661	0.00141	-0.00541	-0.00584	-0.00409
	(30,30)	0.00379	0.00438	0.00383	-0.00113	-0.00656	-0.00618	-0.00416
		0.00336	0.00386	0.00329	-0.00135	-0.00632	-0.00586	-0.00392
	(35,35)	0.00283	0.00320	0.00252	-0.00189	-0.00635	-0.00570	-0.00376
		0.00269	0.00307	0.00252	-0.00146	-0.00559	-0.00508	-0.00336

In each cell the first row represents the average bias of $R_{s,k}$ using the MOM and second row represents average bias of $R_{s,k}$ using the MLE.

Table 2: Average MSE of the simulated estimates of $R_{s,k}$

		(σ_1,σ_2)						
(s,k)	(n,m)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)
	(5,5)	0.02011	0.02755	0.03330	0.02775	0.01296	0.00516	0.00207
		0.01857	0.02576	0.03148	0.02623	0.01202	0.00465	0.00181
	(10,10)	0.00929	0.01389	0.01807	0.01446	0.00512	0.00141	0.00037
		0.00871	0.01306	0.01704	0.01357	0.00470	0.00126	0.00033
	(15,15)	0.00649	0.00991	0.01321	0.01060	0.00353	0.00090	0.00022
		0.00591	0.00906	0.01208	0.00960	0.00311	0.00077	0.00019
(1,3)	(20,20)	0.00461	0.00721	0.00986	0.00795	0.00252	0.00060	0.00014
		0.00415	0.00654	0.00904	0.00735	0.00232	0.00055	0.00013
	(25,25)	0.00381	0.00598	0.00817	0.00643	0.00192	0.00043	0.00009
		0.00347	0.00546	0.00749	0.00589	0.00173	0.00038	0.00008
	(30,30)	0.00303	0.00479	0.00663	0.00529	0.00157	0.00034	0.00007
		0.00278	0.00442	0.00615	0.00492	0.00145	0.00031	0.00007
	(35,35)	0.00258	0.00411	0.00573	0.00458	0.00132	0.00028	0.00006
		0.00234	0.00374	0.00522	0.00416	0.00120	0.00025	0.00005
	(5,5)	0.01198	0.01912	0.02888	0.03374	0.02281	0.01191	0.00575
		0.01089	0.01762	0.02705	0.03197	0.02146	0.01101	0.00520
	(10,10)	0.00494	0.00868	0.01471	0.01853	0.01104	0.00450	0.00161
		0.00461	0.00813	0.01384	0.01748	0.01030	0.00412	0.00145
	(15,15)	0.00336	0.00603	0.01052	0.01366	0.00798	0.00307	0.00104
		0.00305	0.00550	0.00962	0.01247	0.00717	0.00270	0.00089
(2,4)	(20,20)	0.00233	0.00427	0.00768	0.01028	0.00591	0.00217	0.00069
		0.00208	0.00384	0.00697	0.00946	0.00547	0.00200	0.00063
	(25,25)	0.00191	0.00353	0.00637	0.00848	0.00469	0.00164	0.00049
		0.00174	0.00321	0.00582	0.00777	0.00428	0.00148	0.00044
	(30,30)	0.00151	0.00280	0.00511	0.00693	0.00386	0.00133	0.00040
		0.00138	0.00257	0.00472	0.00644	0.00359	0.00123	0.00036
	(35,35)	0.00127	0.00238	0.00439	0.00600	0.00332	0.00112	0.00033
		0.00115	0.00216	0.00399	0.00547	0.00301	0.00101	0.00029

In each cell the first row represents the average MSE of $R_{s,k}$ using the MOM and second row represents average MSE of $R_{s,k}$ using the MLE.

Table 3: Average confidence length of the simulated 95% confidence intervals of $R_{s,k}$

		(σ_1,σ_2)						
(s,k)	(n,m)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)
	(5,5)	0.51137	0.61702	0.69631	0.62210	0.37763	0.20644	0.11255
		0.49107	0.59432	0.67211	0.59907	0.36008	0.19471	0.10505
	(10,10)	0.36638	0.45383	0.52420	0.46425	0.26008	0.12784	0.06239
		0.35086	0.43538	0.50383	0.44615	0.24848	0.12115	0.05865
	(15,15)	0.29764	0.37246	0.43531	0.38721	0.21220	0.10094	0.04771
		0.28525	0.35755	0.41859	0.37200	0.20228	0.09526	0.04459
(1,3)	(20,20)	0.25870	0.32539	0.38200	0.33830	0.18111	0.08383	0.03865
		0.24705	0.31141	0.36661	0.32541	0.17385	0.08012	0.03678
	(25,25)	0.23225	0.29264	0.34402	0.30360	0.16015	0.07282	0.03303
		0.22215	0.28032	0.33011	0.29156	0.15330	0.06935	0.03130
	(30,30)	0.21066	0.26660	0.31531	0.27990	0.14729	0.06646	0.02990
		0.20156	0.25534	0.30236	0.26853	0.14099	0.06341	0.02844
	(35,35)	0.19466	0.24685	0.29272	0.26029	0.13650	0.06122	0.02737
		0.18649	0.23665	0.28080	0.24952	0.13033	0.05817	0.02590
	(5,5)	0.37957	0.49543	0.63356	0.70244	0.54478	0.35494	0.21981
		0.36291	0.47548	0.61050	0.67812	0.52314	0.33795	0.20745
	(10,10)	0.26299	0.35315	0.46760	0.53160	0.39838	0.24122	0.13699
		0.25134	0.33807	0.44870	0.51124	0.38233	0.23022	0.12987
	(15,15)	0.21131	0.28634	0.38425	0.44345	0.33083	0.19598	0.10832
		0.20213	0.27433	0.36895	0.42658	0.31721	0.18656	0.10226
(2,4)	(20,20)	0.18255	0.24863	0.33591	0.38939	0.28720	0.16660	0.09003
		0.17395	0.23734	0.32156	0.37419	0.27623	0.15985	0.08607
	(25,25)	0.16353	0.22312	0.30218	0.35065	0.25670	0.14695	0.07826
		0.15619	0.21337	0.28950	0.33671	0.24637	0.14056	0.07454
	(30,30)	0.14770	0.20222	0.27543	0.32234	0.23678	0.13503	0.07145
		0.14117	0.19345	0.26383	0.30924	0.22707	0.12919	0.06817
	(35,35)	0.13619	0.18679	0.25509	0.29959	0.22010	0.12504	0.06583
		0.13038	0.17892	0.24457	0.28742	0.21076	0.11930	0.06256

In each cell the first row represents the average confidence length of $R_{s,k}$ using the MOM and second row represents average confidence length of $R_{s,k}$ using the MLE.

Table 4: Average coverage probability of the simulated 95% confidence intervals of $R_{s,k}$

		(σ_1,σ_2)						
(s,k)	(n,m)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)
	(5,5)	0.9440	0.9483	0.9510	0.9513	0.9400	0.9400	0.9410
		0.9417	0.9420	0.9503	0.9533	0.9380	0.9367	0.9380
	(10,10)	0.9537	0.9557	0.9533	0.9577	0.9407	0.9370	0.9370
		0.9530	0.9523	0.9507	0.9550	0.9407	0.9373	0.9363
	(15,15)	0.9440	0.9447	0.9430	0.9480	0.9457	0.9407	0.9403
		0.9430	0.9447	0.9427	0.9550	0.9490	0.9423	0.9413
(1,3)	(20,20)	0.9513	0.9500	0.9483	0.9477	0.9417	0.9390	0.9377
		0.9520	0.9487	0.9487	0.9490	0.9443	0.9397	0.9377
	(25,25)	0.9483	0.9493	0.9480	0.9470	0.9473	0.9423	0.9417
		0.9497	0.9483	0.9437	0.9453	0.9450	0.9393	0.9373
	(30,30)	0.9483	0.9470	0.9483	0.9520	0.9517	0.9460	0.9427
		0.9510	0.9500	0.9470	0.9460	0.9507	0.9470	0.9467
	(35,35)	0.9490	0.9490	0.9480	0.9517	0.9547	0.9513	0.9487
		0.9493	0.9510	0.9500	0.9513	0.9553	0.9513	0.9490
	(5,5)	0.9413	0.9437	0.9480	0.9563	0.9460	0.9400	0.9397
		0.9397	0.9407	0.9417	0.9550	0.9437	0.9370	0.9363
	(10,10)	0.9523	0.9533	0.9557	0.9547	0.9500	0.9397	0.9370
		0.9500	0.9523	0.9523	0.9540	0.9497	0.9393	0.9370
	(15,15)	0.9470	0.9453	0.9447	0.9433	0.9497	0.9443	0.9403
		0.9463	0.9440	0.9450	0.9450	0.9533	0.9477	0.9423
(2,4)	(20,20)	0.9533	0.9513	0.9500	0.9490	0.9497	0.9407	0.9387
		0.9533	0.9517	0.9490	0.9483	0.9480	0.9427	0.9397
	(25,25)	0.9490	0.9473	0.9493	0.9470	0.9480	0.9467	0.9423
		0.9480	0.9493	0.9480	0.9430	0.9477	0.9450	0.9393
	(30,30)	0.9480	0.9483	0.9463	0.9497	0.9507	0.9497	0.9460
		0.9513	0.9510	0.9507	0.9470	0.9483	0.9520	0.9470
	(35,35)	0.9490	0.9490	0.9490	0.9493	0.9527	0.9533	0.9513
		0.9493	0.9500	0.9510	0.9513	0.9527	0.9547	0.9513

In each cell the first row represents the average coverage probability of $R_{s,k}$ using the MOM and second row represents average coverage probability of $R_{s,k}$ using the MLE.