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A compromise solution for multi-objective chance constraint capacitated transportation problem

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Abstract. In this paper Goal programming and D_1-Distance approach is used to solve a chance constraint multi-objective capacitated transportation problem. Generally, the capacity of each origin and the demand of each destination are random in nature in transportation problem (TP). The inequality constraints representing supplies and demands are probabilistically described and in model formulation these constraints are converted into equivalent deterministic forms. Then optimum allocation is obtained by using goal programming and D_1-Distance approaches. In order to demonstrate the effectiveness of the proposed approach, an illustrative example is solved.

1. Introduction

Goal programming is a branch of multi-objective optimization, which in turn is a branch of multi-criteria decision analysis, also known as multiple-criteria decision making. It can be thought of as an extension or generalization of linear programming to handle multiple, normally conflicting objective measures. Each of these measures is given a goal or target value to be achieved. Unwanted deviations from this set of target values are then minimized in an achievement function. This can be a vector or a weighted sum dependent on the goal programming variant used. A major strength of GP is its simplicity and ease of use. Goal programming was first used by Charnes et al. (1955), although the actual name first appear in Charnes et al. (1961). Recently, Ali et al. (2011) and Ali et al. (2013) apply goal programming approach in the field of reliability and in sample surveys. Panda et al. (2005) have discussed the EOQ of multi-item inventory problems through nonlinear goal programming.

The TP is one of the subclasses of linear programming problems, in which the objective is to transport various quantities of a single homogenous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. In different fields TP is discussed by many authors among them Hitchcock was first who developed the simplest TP model in 1941 (Hitchcock, 1941). Koopmans (1951), Charnes et al. (1954), Kantorovitch (1960), Haley (1963), Wagner (Wagner, 1959) who made a note on a class of capacitated transportation problem and Pramanik and Banerjee (2012) studied fuzzy goal programming (FGP) approach to multi objective TP with capacity restrictions are some other among them.

Stochastic programming problem was first formulated by Dantizg and Mandansky (1961), who suggested a two stage programming technique for its solutions. Later, Charnes et al. (1961) developed the chance constrained programming technique in which the chance constraints are converted into equivalent deterministic

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non-linear constraints. In 1988, Hassin and Zemel (Hassin and Zemel, 1988) studied probabilistic analysis of the capacitated transportation problem. They showed that asymptotic conditions on the supplies and demands assure a feasible solution to the problem.

In the present paper a chance constraint multi-objective capacitated transportation problem is considered and the optimum compromise solution is obtained by using goal programming and lexicographic goal programming Technique with "Minimum D_1 distances". The obtained compromise solutions compare with Pramanik and Banerjee (2012) approach.

2. Multi-objective capacitated transportation problem

A transportation problem helps us to find out the way in which resources are allocated properly from origins to destinations so that total transportation costs, time, deterioration during transportation etc. would be minimal. We consider p sources (origins) O_i $(i = 1, 2, \ldots, p)$ and q destinations D_j $(j = 1, 2, \ldots, q)$. At each source O_i $(i = 1, 2, \ldots, p)$, let a_i be the amount of product to be shipped to the q destinations D_j in order to satisfy the demand b_j $(j = 1, 2, \ldots, q)$ there. In many practical problems, a_i and b_j cannot be deterministically provided. Here, a_i, b_j are considered as random variables with known distribution. In addition, there exists a penalty c^k_{ij} associated with transporting a unit of product from source O_i to destination D_j for the k^{th} criterion. In general, c^k_{ij} denotes the transportation costs, delivery time, damage charges (loss of quality and quantity of transported items), under used capacity, etc. Let x_{ij} be the variable that represents the unknown quantity transported from i^{th} origin to j^{th} destination. Since, we are interested in capacitated TP, there are limitations on the amount of resources allocated in different cells. Let r_{ij} be the maximum amount of quantity transported from i^{th} source to j^{th} destination, i.e. $x_{ij} \leq r_{ij}$. This restriction is called the capacitated restriction on the route i to j.

Considering k penalty criteria, the mathematical model for MOCTP with chance constraints can be written as:

$$\left.\begin{aligned}
&Min\ Z^k = \sum_{i=1}^{p}\sum_{j=1}^{q} c^k_{ij} x_{ij}, \quad k = 1, 2, \ldots, K \\
&Subject\ to\ \ Prob\left(\sum_{j=1}^{q} x_{ij} \leq a_i\right) \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, p \\
&Prob\left(\sum_{i=1}^{p} x_{ij} \geq b_j\right) \geq 1 - \beta_j, \quad j = 1, 2, \ldots, q \\
&0 \leq x_{ij} \leq r_{ij} \\
&0 < \alpha_i < 1,\ 0 < \beta_j < 1
\end{aligned}\right\}$$

Here, α_i, β_j are the known confidence levels for the constraints and the TP is unbalanced TP. Then the model reduced to deterministic multi-objective transportation problem as follows (see Pramanik and Banerjee (2012)):

$$\left.\begin{aligned}
&Min\ Z^k = \sum_{i=1}^{p}\sum_{j=1}^{q} c^k_{ij} x_{ij}, \quad k = 1, 2, \ldots, K \\
&Subject\ to\ \ \sum_{j=1}^{q} x_{ij} \leq E(a_i) + \Phi(\alpha_i)\sqrt{var(a_i)} \\
&\sum_{i=1}^{p} x_{ij} \geq E(b_j) - \Phi(\beta_j)\sqrt{var(b_j)} \\
&0 \leq x_{ij} \leq r_{ij},\ i = 1, 2, \ldots, p\ \ and\ \ j = 1, 2, \ldots, q
\end{aligned}\right\}$$

3. Goal programming formulation of chance constraint multi-objective capacitated transportation problem (CCMOCTP)

To solve the following MOTP using goal programming, we first solve each objective subject to the system constraints separately

$$\left.\begin{array}{l} Min\ Z^k = \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}^k x_{ij}, \quad k = 1, 2, \ldots, K \\ \\ Subject\ to\ \ \sum_{j=1}^{q} x_{ij} \le E(a_i) + \Phi(\alpha_i)\sqrt{var(a_i)} \\ \\ \sum_{i=1}^{p} x_{ij} \ge E(b_j) - \Phi(\beta_j)\sqrt{var(b_j)} \\ \\ 0 \le x_{ij} \le r_{ij},\ i = 1, 2, \ldots, p\ \ and\ \ j = 1, 2, \ldots, q \end{array}\right\}$$

Let Z^{k^*} be the optimum value of Z^k. Further, let $\widetilde{Z}^k = \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}^k x_{ij}$ as the optimal value under compromise solution. Obviously, $\widetilde{Z}^k \ge Z^{k^*}$ or $\widetilde{Z}^k - Z^{k^*} \ge 0$; $k = 1, 2, \ldots, K$.

A reasonable criterion to work out a compromise allocation may be to "minimize the sum of increases in the objective functions, $Z^k, k = 1, 2, \ldots, K$ due to the use of the compromise solution". Find x_{ij} such that the increase in value of the objectives due to the use of a compromise allocation,x_{ij}, instead of its individual optimum solution, should not greater than $x_k, k = 1, 2, \ldots, K$, where $x_k \ge 0, k = 1, 2, \ldots, K$ are the unknown goal variables.

To achieve these goals x_{ij} must satisfy $\widetilde{Z}^k - Z^{k^*} \le x_k$; $k = 1, 2, \ldots, K$, or $\widetilde{Z}^k - x_k \le Z^{k^*}$ and $\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}^k x_{ij} - x_k \le Z^{k*}$. The value of $\sum_{k=1}^{K} x_k$ will give us total increase in the objectives (i.e. cost, time & damage) by not using the individual optimum allocations. This suggests the following Goal Programming Problem (GPP) to solve:

$$\left.\begin{array}{l} Minimize\ \ \sum_{k=1}^{K} x_k \\ \\ Subject\ to\ \ \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij}^k x_{ij} - x_k \le Z^{k^*}, \quad k = 1, 2, \ldots, K \\ \\ \sum_{j=1}^{q} x_{ij} \le E(a_i) + \Phi(\alpha_i)\sqrt{var(a_i)} \\ \\ \sum_{i=1}^{p} x_{ij} \ge E(b_j) - \Phi(\beta_j)\sqrt{var(b_j)} \\ \\ 0 \le x_{ij} \le r_{ij},\ i = 1, 2, \ldots, p\ \ and\ \ j = 1, 2, \ldots, q \end{array}\right\}$$

The GPP (5) may be solved by using the optimization software LINGO (LINGO-User's Guide, 2001). For more information one can visit the site: http://www.lindo.com.

4. D_1-distance method

In this method the priorities are given to the objectives one after the other and a set of solutions is obtained. Out of these solutions, an ideal solution is identified as follows:

$$x_{ij}^* = \{min(x_{11}^{(1)}, x_{11}^{(2)}), min(x_{22}^{(1)}, x_{22}^{(2)}, \ldots, min(x_{pq}^{(1)}, x_{pq}^{(2)})\} = \{x_{11}^*, x_{22}^*, \ldots, x_{pq}^*\}.$$

A general procedure with P objectives is the following. As explained above, we will obtain $P!$ different solutions by solving the $P!$ problems arising for $P!$ different priority structures.

Let $x_{ij}^{(r)} = \{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}$, $1 \leq r \leq P!$ be the $P!$ number of solutions obtained by giving priorities to P objective functions. Let $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$ be the ideal solution. But in practice ideal solution can never be achieved. The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context. The D_1-distances of different solutions from the ideal solution defined in (1) below are then calculated. The solution corresponding to the minimum D_1-distance gives the best compromise solution.

Now

$$(D_1)^r = \sum_{i=1}^{p} \sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}|$$

is defined as the D_1-distance from the ideal solution $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$, to the r^{th} solution $\{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}$, $1 \leq r \leq P!$. Therefore

$$(D_1)_{opt} = min_{1 \leq r \leq P!} (D_1)^r = min_{1 \leq r \leq P!} \sum_{i=1}^{p} \sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}| \tag{1}$$

Let the minimum be attained for $r = t$. Then $\{x_{11}^{(t)}, x_{22}^{(t)}, \ldots, x_{pq}^{(t)}\}$ is the best compromise solution of the problem.

5. Illustrative Example

To demonstrate the potentiality of the proposed models, we consider the following example. Here, we consider three origins and three destinations. The TP cost, time and the damage charges (both quality and quantity damage) during the transportation are represented by three square matrices of order three. The matrices are given bellow:

Cost matrix: $\begin{bmatrix} 3 & 4 & 13 \\ 12 & 14 & 7 \\ 15 & 10 & 8 \end{bmatrix}$

Time matrix: $\begin{bmatrix} 9 & 1 & 3 \\ 2 & 4 & 6 \\ 8 & 12 & 10 \end{bmatrix}$

Damage charge: $\begin{bmatrix} 8 & 9 & 11 \\ 3 & 4 & 7 \\ 2 & 1 & 8 \end{bmatrix}$

Then the objective functions can be represented by

$$Min \;\; Z_1 = (3x_{11} + 4x_{12} + 13x_{13}) + (12x_{21} + 14x_{22} + 7x_{23}) + (15x_{31} + 10x_{32} + 8x_{33})$$

$$Min \;\; Z_2 = (9x_{11} + x_{12} + 3x_{13}) + (2x_{21} + 4x_{22} + 6x_{23}) + (8x_{31} + 12x_{32} + 10x_{33})$$

$$Min \;\; Z_3 = (8x_{11} + 9x_{12} + 11x_{13}) + (3x_{21} + 4x_{22} + 7x_{23}) + (2x_{31} + x_{32} + 6x_{33})$$

Subject to

$$Prob\left(\sum_{j=1}^{3} x_{1j} \leq a_1\right) \geq 1 - \alpha_1, \; Prob\left(\sum_{j=1}^{3} x_{2j} \leq a_2\right) \geq 1 - \alpha_2, \; Prob\left(\sum_{j=1}^{3} x_{3j} \leq a_3\right) \geq 1 - \alpha_3$$

$$Prob\left(\sum_{i=1}^{3} x_{i1} \geq b_1\right) \geq 1 - \beta_1, \; Prob\left(\sum_{j=1}^{3} x_{1j} \geq b_2\right) \geq 1 - \beta_2, \; Prob\left(\sum_{j=1}^{3} x_{1j} \geq b_3\right) \geq 1 - \beta_3.$$

The capacitated constraints are: $0 \le x_{11} \le 6$, $0 \le x_{12} \le 7$, $0 \le x_{13} \le 13$, $0 \le x_{21} \le 6$, $0 \le x_{22} \le 2$, $0 \le x_{23} \le 13$, $0 \le x_{31} \le 4$, $0 \le x_{32} \le 7$, $0 \le x_{33} \le 14$. The mean, variance and the confidence levels are described below:

$$E(a_1) = 12, var(a_1) = 9, \alpha_1 = 0.01$$

$$E(a_2) = 15, var(a_2) = 4, \alpha_2 = 0.02$$

$$E(a_3) = 20, var(a_3) = 7, \alpha_3 = 0.03$$

$$E(b_1) = 9, var(b_1) = 2, \beta_1 = 0.01$$

$$E(b_2) = 13, var(b_2) = 8, \beta_2 = 0.02$$

$$E(b_3) = 21, var(b_3) = 16, \beta_3 = 0.03.$$

Using these mean, variance and confidence levels we get our required deterministic constraints as:

$$\sum_{j=1}^{3} x_{1j} \le 18.975,\ \sum_{j=1}^{3} x_{2j} \le 19.11,\ \sum_{j=1}^{3} x_{3j} \le 24.987$$

$$\sum_{i=1}^{3} x_{i1} \ge 5.7119,\ \sum_{i=1}^{3} x_{2i} \ge 7.1876,\ \sum_{i=1}^{3} x_{i3} \ge 13.46.$$

5.1. A compromise solution using Goal programming

$$Min\ \ Z_1 = (3x_{11} + 4x_{12} + 13x_{13}) + (12x_{21} + 14x_{22} + 7x_{23}) + (15x_{31} + 10x_{32} + 8x_{33})$$

$$Subject\ to$$

$$\sum_{j=1}^{3} x_{1j} \le 18.975,\ \sum_{j=1}^{3} x_{2j} \le 19.11,\ \sum_{j=1}^{3} x_{3j} \le 24.987$$

$$\sum_{i=1}^{3} x_{i1} \ge 5.7119,\ \sum_{i=1}^{3} x_{2i} \ge 7.1876,\ \sum_{i=1}^{3} x_{i3} \ge 13.46,$$

$$0 \le x_{11} \le 6,\ 0 \le x_{12} \le 7,\ 0 \le x_{13} \le 13,\ 0 \le x_{21} \le 6,\ 0 \le x_{22} \le 2,\ 0 \le x_{23} \le 13,\ 0 \le x_{31} \le 4,$$

$$0 \le x_{32} \le 7,\ 0 \le x_{33} \le 14.$$

The optimum solution provide by LINGO is: $x_{11}^* = 5.7119$, $x_{12}^* = 7$, $x_{13}^* = 0$, $x_{21}^* = 0$, $x_{22}^* = 0$, $x_{23}^* = 13$, $x_{31}^* = 0$, $x_{32}^* = 0.1876$, $x_{33}^* = 0.4600$ with $Z_1^* = 141.6917$.

Similarly, using LINGO we obtain the optimum solution for second and third objective as follows. The optimum solution of second objective provide by LINGO is: $x_{11}^* = 0$, $x_{12}^* = 5.9750$, $x_{13}^* = 13$, $x_{21}^* = 5.7119$, $x_{22}^* = 1.2126$, $x_{23}^* = 0.4600$, $x_{31}^* = 0$, $x_{32}^* = 0$, $x_{33}^* = 0$ with $Z_2^* = 64.00920$.

The optimum solution of third objective provide by LINGO is: $x_{11}^* = 0$, $x_{12}^* = 0$, $x_{13}^* = 0$, $x_{21}^* = 1.71190$, $x_{22}^* = 0.18760$, $x_{23}^* = 0$, $x_{31}^* = 4$, $x_{32}^* = 7$, $x_{33}^* = 13.4600$ with $Z_3^* = 101.6461$.

After finding the optimum values of Z_j^*, $j = 1, 2, 3$, the GPP can be written as:

$$\left.\begin{array}{l}
Minimize = x_1 + x_2 + x_3 \\
Subject\ to \\
((3x_{11} + 4x_{12} + 13x_{13}) + (12x_{21} + 14x_{22} + 7x_{23}) + (15x_{31} + 10x_{32} + 8x_{33})) - x_1 \leq 141.6917 \\
((9x_{11} + x_{12} + 3x_{13}) + (2x_{21} + 4x_{22} + 6x_{23}) + (8x_{31} + 12x_{32} + 10x_{33})) - x_2 \leq 64.00920 \\
((8x_{11} + 9x_{12} + 11x_{13}) + (3x_{21} + 4x_{22} + 7x_{23}) + (2x_{31} + x_{32} + 6x_{33})) - x_3 \leq 101.6461 \\
\sum_{j=1}^{3} x_{1j} \leq 18.975,\ \sum_{j=1}^{3} x_{2j} \leq 19.11,\ \sum_{j=1}^{3} x_{3j} \leq 24.987 \\
\sum_{i=1}^{3} x_{i1} \geq 5.7119,\ \sum_{i=1}^{3} x_{2i} \geq 7.1876,\ \sum_{i=1}^{3} x_{i3} \geq 13.46, \\
0 \leq x_{11} \leq 6,\ 0 \leq x_{12} \leq 7,\ 0 \leq x_{13} \leq 13,\ 0 \leq x_{21} \leq 6,\ 0 \leq x_{22} \leq 2,\ 0 \leq x_{23} \leq 13, \\
0 \leq x_{31} \leq 4,\ 0 \leq x_{32} \leq 7,\ 0 \leq x_{33} \leq 14 \\
x_1, x_2, x_3 \geq 0.
\end{array}\right\}$$

Using the LINGO software, the optimum compromise allocation is found to be $x_{11}^* = 0$, $x_{12}^* = 7$, $x_{13}^* = 0$, $x_{21}^* = 5.7119$, $x_{22}^* = 0$, $x_{23}^* = 13$, $x_{31}^* = 0$, $x_{32}^* = 0$, $x_{33}^* = 0.46$ with $x_1 = 52.61750$, $x_2 = 37.76500$, $x_3 = 73$.

5.2. A compromise solution using D_1-distance

Since we have three objectives of minimizing transportation cost, time and damage charges in our example, so we have to solve $3! = 6$ problems according to the priority. The solutions obtained by giving priority to each of the objectives one by one is given below on the Tables 1 and 2 (all the problems are solved by optimization software LINGO).

The minimum distance is 11.422 corresponding to the priority structure $(Z_1^{(1)}, Z_2^{(2)}, Z_3^{(3)})$. Therefore, the optimum allocation is given by $x_{11}^* = 5.7119$, $x_{12}^* = 2.5975$, $x_{13}^* = 0.1381$, $x_{21}^* = 0$, $x_{22}^* = 0.0879$, $x_{23}^* = 2.4651$, $x_{31}^* = 0$, $x_{32}^* = 0.0997$, $x_{33}^* = 0.3218$ with $Z_1^* = 51.37870$, $Z_2^* = 73.97550$, $Z_3^* = 90.22960$.

6. Another criteria given by Pramanik & Banerjee

Pramanik and Banerjee (2012) define three Fuzzy goal programming models to solve CCMOCTP and to choose compromise optimum allocation they define distance function and obtain the optimum allocation as follows: $x_{11}^* = 0.0619$, $x_{12}^* = 7$, $x_{13}^* = 0$, $x_{21}^* = 5.650$, $x_{22}^* = 0$, $x_{23}^* = 13.46$, $x_{31}^* = 0$, $x_{32}^* = 0.1876$, $x_{33}^* = 0$ with $Z_1^* = 192.0817$, $Z_2^* = 101.8683$, $Z_3^* = 174.8528$.

7. Conclusion

In the present manuscript we proposed two approaches viz. Goal programming and D_1-distance to obtain optimum compromise allocation of CCMOCTP and compare with Pramanik & Banerjee's approach (P & B). In this comparative study we find out that D_1-distance gives the best optimum solution of the given problem as compared to the other two which is clearly shown in the Table 3.

Table 1: Solutions

Priority structure	Objective value	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
$Z_1^{(1)}, Z_2^{(2)}, Z_3^{(3)}$	1.042421	5.7119	7	0.1381	**0**	0.0879	13	**0**	0.0997	0.3218
$Z_1^{(1)}, Z_3^{(2)}, Z_2^{(3)}$	12.72942	5.7119	6.5523	0	0	0	13	0	0.6353	0.4600
$Z_2^{(1)}, Z_1^{(2)}, Z_3^{(3)}$	87.78209	0	5.4760	1.7735	5.7119	1.7116	11.6865	0	0	0
$Z_2^{(1)}, Z_3^{(2)}, Z_1^{(3)}$	74.76205	5.7119	7	1.4507	0	0.1876	12.0094	0	0	0
$Z_3^{(1)}, Z_1^{(2)}, Z_2^{(3)}$	90.59747	0	5.1876	2.9251	5.7119	2	10.5349	0	0	0
$Z_3^{(1)}, Z_2^{(2)}, Z_1^{(3)}$	88.6581	5.7119	4.4025	0	0	0	13	0	2.7852	0.4600
Ideal solution (x_{ij}^*)		0	4.405	0	0	0	10.5349	0	0	0

Table 2: The D_1-distance from the ideal solutions

Priority structure	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	$(D_1)^r$
$Z_1^{(1)}, Z_2^{(2)}, Z_3^{(3)}$	5.7119	2.5975	0.1381	0	0.0879	2.4651	0	0.0997	0.3218	11.4220
$Z_1^{(1)}, Z_3^{(2)}, Z_2^{(3)}$	5.7119	2.1498	0	0	0	2.4651	0	0.6353	0.4600	11.4221
$Z_2^{(1)}, Z_1^{(2)}, Z_3^{(3)}$	0	1.0735	1.7735	5.7119	1.7116	1.1516	0	0	0	11.7865
$Z_2^{(1)}, Z_3^{(2)}, Z_1^{(3)}$	5.7119	2.5975	1.4507	0	0.1876	1.4745	0	0	0	11.4222
$Z_3^{(1)}, Z_1^{(2)}, Z_2^{(3)}$	0	0.7851	2.9251	5.7119	2	0	0	0	0	11.4221
$Z_3^{(1)}, Z_2^{(2)}, Z_1^{(3)}$	5.7119	0	0	0	0	2.4651	0	2.7852	0.4600	11.4222

Table 3: Compromise solutions

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	Cost	Time	Damage Charges
P & B	0.0619	7	0	5.650	0	13.46	0	0.1876	0	192.0817	101.8683	174.8528
Goal Programming	0	7	0	5.7119	0	13	0	0	0.46	191.2228	101.0238	173.8957
D_1-distance	5.7119	2.5975	0.1381	0	0.0879	2.4651	0	0.0997	0.3218	51.37870	73.97550	90.22960

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