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Variable Control Charts Based on Percentiles of Size Biased Lomax Distribution

B Srinivasa Rao†, A N Durgamamba & R Subba Rao

Department of Mathematics & Humanities, R.V.R & J.C College of Engineering, Chowdavaram, Guntur - 522 019, India.
Raghu Institute of Technology, Dakamarri, Vizag - 531162, India. durgamamba@gmail.com
Shree Vishnu Engineering College for Women, Bhimavaram, Andhra Pradesh, India. rsr_vishnu@rediffmail.com

Abstract. A variable quality characteristic is assumed to follow size biased Lomax Distribution. Based on the evaluated percentiles of sample statistic like mean, median, midrange , range and standard deviation , the control limits for the respective control charts are developed . The admissibility and power of the control limits is assessed in comparison with those based on the popular Shewart control limits.

1. Introduction

The well-known Shewart control charts are developed under the assumption that the quality characteristic follows a normal distribution. If $x_1, x_2, .., x_n$ is a collection of observations of size n on a variable quality characteristic of a product and if t_n is a statistic based on this sample, the control limits of Shewart variable control chart are $E(t_n) \pm 3S.E(t_n)$. In quality control studies data is always in small samples only. Since most of the distributions tend to normal distribution, it is taken as an alternative solution for all the distributions because of its central limit theorem. And if the data is assumed to follow normal distribution, the commonly used constants are Shewart constants. Even if a skewed data which follows size biased lomax distribution it is not advisable to apply the Shewart constants. An alternative procedure is to be adopted . Therefore if the population is not normal there is a need to develop a separate procedure for the construction of control limits. In this paper we assume that the quality variate follows size biased Lomax model and develop control limits for such a data on par with the presently available control limits. If a process quality characteristic is assumed to follow size biased Lomax distribution the online process of such a quality can be controlled through the theory of size biased Lomax distribution. In the absence of any such specification of the population model we generally use the normal distribution and the associated constants available in all standard text books of statistical quality control. However, normality is only an assumption that is rarely verified and found to be true. Unless the sample is very large in size this assumption may not be taken for granted without proper goodness of fit test procedure. At the same time central limit theorem cannot be made use of, because central limit theorem gives only asymptotic normality for any statistic. Therefore, if a distribution other than normal is a suitable model for a quality variate, separate procedures are to be developed. We present the construction of quality control charts when the process variate is assumed to follow size biased Lomax distribution.

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†Corresponding author
Email address: boyapatisrinu@yahoo.com (B Srinivasa Rao†)

The probability density function (pdf) of size biased Lomax distribution (SBLD) is given by

$$f(x) = \frac{\alpha(\alpha-1)x}{\sigma^2}(1+\frac{x}{\sigma})^{(-\alpha+1)}, \qquad x>0, \alpha>0, \sigma>0 \tag{1}$$

Its cumulative distribution function (cdf) is

$$F(x) = 1-(1+\frac{\alpha x}{\sigma})(1+\frac{x}{\sigma})^{-\alpha}, \qquad x>0, \alpha>0, \sigma>0 \tag{2}$$

Size biased Lomax distribution (SBLD) is a skewed , unimodal distribution on the positive real line. The distributional properties are

$$mean = \frac{2\sigma}{\alpha-2}, \qquad \alpha>2 \tag{3}$$

$$mode = \frac{\sigma}{\alpha} \tag{4}$$

$$median = \frac{\sigma(5\alpha-2)}{3\alpha(\alpha-2)}, \qquad \alpha>2 \tag{5}$$

$$variance = \frac{2\alpha\sigma^2}{(\alpha-2)^2(\alpha-3)}, \qquad \alpha>3 \tag{6}$$

Skewed distributions to develop statistical quality control methods are attempted by many authors. Some of them are Edgeman(1989) (3)-Inverse Gaussion Distribution, Gonzalez and Viles(2000) (4)-Gamma Distribution, Kantam and Sriram(2001) (5)-Gamma Distribution, Chan and Cui (2003) (2) have developed control chart constants for skewed distributions where the constants are dependent on the coefficient of skewness of the distribution, Kantam *et al*(2006) (6)-Log logistic Distribution, Betul and Yaziki(2006) (1)-Burr Distribution , Subba Rao and Kantam(2008) (10)-Double exponential distribution, Kantam and Srinivasa Rao(2010) (7)-control charts for process variate, Srinivasa Rao and Sarath Babu (2012) (8)-Linear failure rate distribution, Srinivasa Rao and Kantam (2012) (9)-Half logistic distribution and references there in. SBLD is another situation of skewed distribution that was not paid much attention with respect to development of control charts. At the same time it is one of the probability models applicable for life testing and reliability studies. Accordingly, if a lifetime data is considered as a quality data , development of control charts for the same is desirable for the use by practitioners. Since SBLD is a skewed distribution , this paper makes an attempt to study in a comparative manner. An attempt is made in this paper to address this problem and solve it to the extent possible. The rest of the paper is organized as follows. The basic theory and the development of control charts for the statistics- mean, median, midrange , range and standard deviation are presented in Section 2. The comparative study of the developed control limits in relation to the Shewart limits is given in Section 3. Summary and conclusions are given in Section 4.

2. Control chart constants through percentiles

2.1. $\overline{X}$ − *chart*

Let $x_1, x_2, ..x_n$ be a random sample of size n supposed to have been drawn from SBLD with α=6 and σ=1.(Since the mean, mode, median and variance exist for these values under the stated conditions in equations 1.3, 1.4 1.5 and 1.6) If this is considered as a subgroup of an industrial process data with a targeted population average, under repeated sampling the statistic $\bar{x}$ gives whether the process average is around the targeted mean or not. Statistically speaking, we have to find the 'most probable' limits within which $\bar{x}$ falls. Here the phrase 'most probable' is a relative concept which is to be considered in the population sense. As the existing procedures are for normal distribution only, the concept of 3σ limits is taken as the 'most probable' limits. It is well known that 3σ limits of normal distribution include 99.73% of probability. Hence, we have to search two limits of the sampling distribution of sample mean in SBLD such that the probability content of those limits is 0.9973. Symbolically we have to find L, U such that

$$P(L \leq \bar{x} \leq U) = 0.9973 \tag{7}$$

Where $\bar{x}$ is the mean of sample size n. Taking the equi-tailed concept L,U are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of $\bar{x}$. We resorted to the empirical sampling distribution of $\bar{x}$ through simulation thereby computing its percentiles. These are given in the table 2.1.1.

Table 2.1.1: Percentiles of Mean in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	2.0149	1.2292	0.9210	0.7005	0.1312	0.1254	0.1154	0.1073
3	1.7384	1.0178	0.8106	0.6467	0.1417	0.1336	0.1253	0.1146
4	1.4670	0.8863	0.7261	0.5995	0.1490	0.1418	0.1331	0.1214
5	1.2349	0.8363	0.6706	0.5695	0.1541	0.1471	0.1386	0.1282
6	1.0671	0.7591	0.6285	0.5455	0.1589	0.1518	0.1431	0.1308
7	1.0292	0.7125	0.6010	0.5163	0.1617	0.1543	0.1474	0.1362
8	0.9640	0.6830	0.5738	0.5051	0.1652	0.1572	0.1506	0.1381
9	0.9328	0.6478	0.5629	0.4956	0.1681	0.1608	0.1536	0.1426
10	0.8771	0.6339	0.5451	0.4865	0.1713	0.1641	0.1565	0.1446

The percentiles in the above table are used in the following manner to get the control limits for sample mean. From the distribution of $\bar{x}$, consider

$$P(Z_{0.00135} \leq \bar{x} \leq Z_{0.99865}) = 0.9973 \tag{8}$$

But $\bar{x}$ of sampling distribution when α=6 and $\sigma = 1$ is 0.5 for SBLD.
From equation(8) over repeated sampling, for the i^{th} subgroup mean we can have

$$P(Z_{0.00135}\frac{\bar{\bar{x}}}{0.5} \leq \bar{x}_i \leq Z_{0.99865}\frac{\bar{\bar{x}}}{0.5}) = 0.9973 \tag{9}$$

this can be written as

$$P({}^{*}_{2p}\times\bar{\bar{x}} \leq \bar{x}_i \leq^{**}_{2p} \times\bar{\bar{x}}) = 0.9973 \tag{10}$$

where $\bar{\bar{x}}$ is grand mean, $\bar{x}_i$ is i^{th} subgroup mean, $A^{*}_{2p} = \frac{Z_{0.00135}}{0.5}, A^{**}_{2p} = \frac{Z_{0.99865}}{0.5}$. Thus ${}^{*}_{2p},{}^{**}_{2p}$ are the percentile constants of $\bar{x}$ chart for SBLD are given in table 2.1.2.

Table 2.1.2 Percentile constants of $\bar{x}$-chart		
n	A^{*}_{2p}	A^{**}_{2p}
2	0.0611	1.1472
3	0.0655	0.9936
4	0.0690	0.8338
5	0.0730	0.7031
6	0.0743	0.6067
7	0.0776	0.5864
8	0.0788	0.5504
9	0.0814	0.5328
10	0.0827	0.5020

2.2. ***Median-chart***

We have to search two limits of the sampling distribution of sample median in SBLD such that the probability content of these limits is 0.9973. Symbolically, we have to find L,U such that

$$P(L \leq m \leq U) = 0.9973 \tag{11}$$

where m is the median of sample size n. Through simulation, the percentiles observed are given in the table 2.2.1.

Table 2.2.1: Percentiles of Median in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	2.0149	1.2292	0.9210	0.7005	0.1312	0.1224	0.1154	0.1073
3	1.9986	0.7809	0.5996	0.4531	0.1270	0.1190	0.1134	0.1067
4	1.8745	0.6333	0.5130	0.4107	0.1387	0.1298	0.1228	0.1131
5	1.6577	0.5546	0.4204	0.3007	0.1365	0.1288	0.1211	0.1123
6	1.5622	0.4724	0.3789	0.3117	0.1445	0.1370	0.1295	0.1168
7	1.4528	0.4438	0.3158	0.2392	0.1421	0.1344	0.1272	0.1174
8	1.3679	0.4003	0.3191	0.2616	0.1480	0.1406	0.1335	0.1209
9	1.2524	0.3641	0.2704	0.2354	0.1469	0.1397	0.1328	0.1223
10	1.1149	0.3384	0.2682	0.2352	0.1507	0.1440	0.1375	0.1267

The percentiles in the above table are used in the following manner to get the control limits for median. From the distribution of m, consider

$$P(Z_{0.00135} \leq m \leq Z_{0.99865}) = 0.9973 \tag{12}$$

But median of sampling distribution when α=6 and $\sigma = 1$ is 0.3889 for SBLD.
From equation(12) over repeated sampling, for the i^{th} subgroup median we can have

$$P(Z_{0.00135}\frac{\overline{m}}{0.3889} \leq m_i \leq Z_{0.99865}\frac{\overline{m}}{0.3889}) \tag{13}$$

This can be written as

$$P(A^{*}_{7p}\overline{m} \leq \overline{m_i} \leq A^{**}_{7p}\overline{m}) = 0.9973 \tag{14}$$

where $\overline{m}$ is mean of subgroup medians. Thus $A^{*}_{7p} = \frac{Z_{0.00135}}{0.3889}, A^{**}_{7p} = \frac{Z_{0.99865}}{0.3889}$ are the percentile constants of median chart and are given in table 2.2.2.

Table 2.2.2
Percentile constants of
Median-chart

n	A^{*}_{7p}	A^{**}_{7p}
2	0.0785	1.4750
3	0.0595	1.1157
4	0.0630	1.0454
5	0.0582	0.8575
6	0.0604	0.8086
7	0.0591	0.7310
8	0.0610	0.6901
9	0.0607	0.6225
10	0.0630	0.5544

2.3. *Midrange-chart*

We have to search two limits of the sampling distribution of sample midrange in SBLD such that the probability content of these limits is 0.9973. Symbolically, we have to find L,U such that

$$P(L \leq M \leq U) = 0.9973 \tag{15}$$

where M is the midrange of sample size n . Through simulation, the percentiles observed are given in the table 2.3.1.

Table 2.3.1: Percentiles of Midrange in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	2.0149	1.2292	0.9210	0.7005	0.1312	0.1224	0.1154	0.1073
3	2.4489	1.2860	0.9935	0.7780	0.1428	0.1352	0.1258	0.1152
4	2.5074	1.3730	1.0540	0.8508	0.1514	0.1439	0.1346	0.1216
5	2.5244	1.4669	1.1310	0.9097	0.1583	0.1509	0.1431	0.1306
6	2.5308	1.5215	1.1952	0.9633	0.1631	0.1564	0.1482	0.1354
7	2.5520	1.5537	1.2458	1.0077	0.1658	0.1600	0.1531	0.1393
8	2.6194	1.6088	1.2694	1.0396	0.1683	0.1624	0.1556	0.1406
9	2.7330	1.6761	1.3224	1.0814	0.1708	0.1657	0.1587	0.1447
10	2.8833	1.7508	1.3776	1.1292	0.1734	0.1680	0.1626	0.1510

The percentiles from the above table are used in the following manner to get the control limits for midrange. From the distribution of M, consider

$$P(Z_{0.00135} \leq M \leq Z_{0.99865}) = 0.9973 \tag{16}$$

The median value of SBLD is not mathematically tractable , therefore $\alpha_{(1)}$ is calculated by $F(x) = \frac{1}{n+1}$ and $\alpha_{(n)}$ by $F(x) = \frac{n}{n+1}$.
From equation (16) for i^{th} subgroup midrange we can have,

$$P(Z_{0.00135} \frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(n)}}{2}} \leq M_i \leq Z_{0.99865} \frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(n)}}{2}}) \tag{17}$$

This can be written as

$$P(A^*_{4p}\overline{M} \leq M_i \leq A^{**}_{4p}\overline{M}) = 0.9973 \tag{18}$$

where $\overline{M}$ is mean of midranges. Thus $A^*_{4p} = \frac{2Z_{0.00135}}{\alpha_{(1)}+\alpha_{(n)}}, A^{**}_{4p} = \frac{2Z_{0.99865}}{\alpha_{(1)}+\alpha_{(n)}}$ are the percentile constants of midrange chart for SBLD process data given in table 2.3.2.

Table 2.3.2
Percentile constants of Midrange-chart

n	A^*_{4p}	A^{**}_{4p}
2	0.0799	1.5016
3	0.0888	1.8895
4	0.0955	1.9695
5	0.1050	2.0301
6	0.1108	2.0718
7	0.1157	2.1078
8	0.1187	2.2124
9	0.1235	2.3339
10	0.1305	2.4926

2.4. ***R-chart***

We have to search two limits of the sampling distribution of sample range in SBLD such that the probability content of these limits is 0.9973. Symbolically, we have to find L, U such that

$$P(L \leq R \leq U) = 0.9973 \tag{19}$$

where R is the range of sample of size n. Through simulation, the percentiles observed are given in the table 2.4.1.

Table 2.4.1: Percentiles of Range in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	3.5877	1.8186	1.2776	0.9517	0.0061	0.0032	0.0012	0.0002
3	4.6256	2.2651	1.6495	1.2111	0.0274	0.0195	0.0133	0.0042
4	4.6552	2.4481	1.8101	1.4014	0.0483	0.0382	0.0278	0.0123
5	4.7210	2.6407	1.9985	1.5442	0.0668	0.0557	0.0422	0.0246
6	4.8056	2.7761	2.1301	1.6624	0.0807	0.0688	0.0560	0.0375
7	4.8227	2.8440	2.2375	1.7637	0.0929	0.0815	0.0698	0.0471
8	5.0169	2.9787	2.2914	1.8384	0.1028	0.0890	0.0777	0.0524
9	5.2300	3.1282	2.3819	1.9198	0.1109	0.0979	0.0845	0.0644
10	5.5353	3.2811	2.5174	2.0122	0.1187	0.1065	0.0941	0.0756

The percentiles from the above table are used in the following manner to get the control limits for sample range. From distribution of R, consider

$$P(Z_{0.00135} \leq R \leq Z_{0.99865}) = 0.9973 \tag{20}$$

From equation(20), for the $\imath^{th}$ subgroup range we can have

$$P(Z_{0.00135}\frac{\overline{R}}{\alpha_{(n)} - \alpha_{(1)}} \leq R_i \leq Z_{0.99865}\frac{\overline{R}}{\alpha_{(n)} - \alpha_{(1)}}) = 0.9973 \tag{21}$$

This can be written as

$$P(D_{3p}^{*}\overline{R} \leq R_i \leq D_{4p}^{**}\overline{R}) = 0.9973 \tag{22}$$

where $\overline{R}$ is mean of ranges, R_i is i^{th} subgroup range. Thus $D_{3p}^{*} = \frac{Z_{0.00135}}{\alpha_{(n)}-\alpha_{(1)}}, D_{4p}^{**} = \frac{Z_{0.99865}}{\alpha_{(n)}-\alpha_{(1)}}$ are the percentile constants of R chart for SBLD process data given in table 2.4.2.

Table 2.4.2
Percentile constants of Range-chart

n	D_{3p}^{*}	D_{4p}^{**}
2	0.0001	2.9295
3	0.0032	3.5708
4	0.0092	3.4918
5	0.0185	3.5524
6	0.0284	3.6487
7	0.0361	3.6960
8	0.0407	3.8960
9	0.0504	4.0978
10	0.0599	4.3875

2.5. σ -chart

We have to search two limits of the sampling distribution of sample standard deviation in SBLD such that the probability content of these limits is 0.9973. Symbolically, we have to find L, U such that

$$P(L \leq s \leq U) = 0.9973 \tag{23}$$

where s is the standard deviation of sample of size n. Through simulation the percentiles observed are given in the table 2.5.1.

Table 2.5.1: Percentiles of standard deviation in SBLD

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	2.7938	0.9093	0.6388	0.4758	0.0031	0.0016	0.0006	0.0001
3	2.1727	1.0427	0.7506	0.5500	0.0117	0.0083	0.0057	0.0018
4	1.9549	1.0215	0.7536	0.5788	0.0192	0.0149	0.0110	0.0047
5	1.8205	1.0117	0.7778	0.6003	0.0246	0.0204	0.0155	0.0088
6	1.7633	0.9941	0.7760	0.6069	0.0281	0.0240	0.0198	0.0129
7	1.6741	0.9828	0.7650	0.6021	0.0316	0.0277	0.0235	0.0158
8	1.6093	0.9675	0.7395	0.6046	0.0343	0.0295	0.0251	0.0178
9	1.6108	0.9616	0.7402	0.5999	0.0362	0.0316	0.0270	0.0203
10	1.6104	0.9827	0.7486	0.6139	0.0386	0.0338	0.0302	0.0242

The percentiles from the above table are used in the following manner to get the control limits for sample standard deviation. From distribution of s, consider

$$P(Z_{0.00135} \leq s \leq Z_{0.99865}) = 0.9973 \tag{24}$$

But S.D of sampling distribution when α=6 and $\sigma = 1$ is 0.5 for SBLD. From equation(24), for the $\imath^{th}$ subgroup standard deviation we can have

$$P(Z_{0.00135}\frac{\bar{s}}{0.5} \leq s_i \leq Z_{0.99865}\frac{\bar{s}}{0.5}) = 0.9973 \tag{25}$$

This can be written as

$$P(B^*_{3p}\bar{s} \leq s_i \leq B^{**}_{4p}\bar{s}) = 0.9973 \tag{26}$$

where $\bar{s}$ is mean of standard deviations, s_i is i^{th} subgroup standard deviation . Thus $B^*_{3p} = \frac{Z_{0.00135}}{0.5}$, $B^{**}_{4p} = \frac{Z_{0.99865}}{0.5}$ are the constants of standard deviation chart for SBLD process data given in table 2.5.2.

Table 2.5.2
Percentile constants of S.D -chart

n	B^*_{3p}	B^{**}_{4p}
2	0.00002	0.63419
3	0.00055	0.66702
4	0.00163	0.68147
5	0.00336	0.69579
6	0.00526	0.71942
7	0.00678	0.71886
8	0.00797	0.72128
9	0.00939	0.74580
10	0.01154	0.76832

3. Comparitive study

The control chart constants for the statistics mean, median, midrange, range and standard deviation developed in section 2 are based on the population described by SBLD. In order to use this for a data, the data is confirmed to follow SBLD. Therefore the power of the control limits can be assessed through their application for a true SBLD data in relation to the application of Shewart limits . With this back drop we have made this comparative study simulating random samples of size n=2(1)10 from SBLD and calculated the control limits using the constants of section 2 for mean, median, midrange ,range and standard deviation in succession. The number of statistic values that have fallen within the respective control limits is evaluated and is named as SBLD coverage probability. Similar count for control limits using Shewart constants available in quality control manuals are also calculated. These are named as Shewart coverage probability. The coverage probabilities under the two schemes namely true SBLD, Shewart limits are presented in the following tables 3.1, 3.2 , 3.3 , 3.4 and 3.5

Table 3.1 : Coverage Probabilities of Mean-chart

	Shewart limits			Percentile limits		
n	$\overline{\overline{x}} - A_2\bar{R}$	$\overline{\overline{x}} + A_2\bar{R}$	coverage probability	$A^*_{2p} \times \overline{\overline{x}}$	$A^{**}_{2p} \times \overline{\overline{x}}$	coverage probability
2	0	0.7114	**0.9523**	0.0611	1.1472	**0.9873**
3	0	0.6380	**0.9484**	0.0655	0.9936	**0.9887**
4	0	0.5953	**0.9490**	0.0690	0.8338	**0.9856**
5	0	0.5743	**0.9510**	0.0730	0.7031	**0.9793**
6	0.0096	0.5589	**0.9559**	0.0743	0.6067	**0.9711**
7	0.0212	0.5485	**0.9615**	0.0776	0.5864	**0.9716**
8	0.0294	0.5415	**0.9643**	0.0788	0.5504	**0.9680**
9	0.0371	0.5340	**0.9669**	0.0814	0.5428	**0.9703**
10	0.0440	0.5384	**0.9706**	0.0827	0.5520	**0.9785**

Table 3.2 : Coverage Probabilities of Median-chart

	Shewart limits			Percentile limits		
n	$\overline{m} - A_7\bar{R}$	$\overline{m} + A_7\bar{R}$	coverage probability	$A^*_{7p} \times \overline{m}$	$A^{**}_{7p} \times \overline{m}$	coverage probability
2	0	0.7114	**0.9523**	0.0785	1.1472	**0.9953**
3	0	0.5844	**0.9734**	0.0595	1.1157	**0.9978**
4	0	0.5566	**0.9805**	0.0630	1.0454	**0.9995**
5	0	0.5328	**0.9883**	0.0582	0.8575	**0.9987**
6	0	0.5311	**0.9948**	0.0604	0.8086	**0.9997**
7	0	0.5235	**0.9956**	0.0590	0.7310	**0.9997**
8	0	0.5236	**0.9985**	0.0610	0.6901	**0.9997**
9	0	0.5206	**0.9987**	0.0607	0.6225	**0.9998**
10	0	0.5228	**0.9991**	0.0630	0.5544	**0.9992**

Table 3.3 : Coverage Probabilities of Midrange-chart

	Shewart limits			Percentile limits		
n	$\overline{M} - A_4\bar{R}$	$\overline{M} + A_4\bar{R}$	coverage probability	$A^*_{4p} \times \overline{M}$	$A^{**}_{4p} \times \overline{M}$	coverage probability
2	0	0.7893	**0.9627**	0.0799	1.5016	**0.9959**
3	0	0.7560	**0.9456**	0.0888	1.8894	**0.9974**
4	0	0.7049	**0.9188**	0.0955	1.9695	**0.9973**
5	0.0258	0.7405	**0.9126**	0.1050	2.0301	**0.9968**
6	0.0920	0.7323	**0.8917**	0.1108	2.0718	**0.9968**
7	0.1113	0.7670	**0.8895**	0.1157	2.1208	**0.9960**
8	0.1612	0.7695	**0.8532**	0.1187	2.2124	**0.9962**
9	0.1792	0.7985	**0.7848**	0.1235	2.3339	**0.9972**
10	0.2200	0.8035	**0.7331**	0.1305	2.4926	**0.9973**

Table 3.4 : Coverage Probabilities of Range-chart

	Shewart limits			Percentile limits		
n	$D_3\bar{R}$	$D_4\bar{R}$	coverage probability	$D^*_{3p} \times \overline{R}$	$D^{**}_{4p} \times \overline{R}$	coverage probability
2	0	0.7416	**0.9223**	0.00016	2.9295	**0.9966**
3	0	0.8865	**0.9012**	0.00324	3.5708	**0.9968**
4	0	0.9739	**0.8921**	0.00922	3.4918	**0.9963**
5	0	1.0615	**0.8855**	0.01851	3.5524	**0.9956**
6	0	1.1396	**0.8793**	0.02847	3.6487	**0.9958**
7	0.0478	1.2105	**0.8733**	0.03609	3.6960	**0.9950**
8	0.0933	1.2796	**0.8420**	0.04069	3.8960	**0.9948**
9	0.1356	1.3389	**0.7613**	0.05045	4.0978	**0.9954**
10	0.1753	1.3972	**0.7618**	0.05992	4.3875	**0.9959**

Table 3.5 : Coverage Probabilities of S.D-chart

	Shewart limits			Percentile limits		
n	$B_3\bar{s}$	$B_4\bar{s}$	coverage probability	$B^*_{3p} \times \bar{s}$	$B^{**}_{4p} \times \bar{s}$	coverage probability
2	0	0.3708	**0.9223**	0.00002	0.6342	**0.9741**
3	0	0.3941	**0.9000**	0.00055	0.6670	**0.9669**
4	0	0.3949	**0.8880**	0.00163	0.6814	**0.9650**
5	0	0.3992	**0.8796**	0.00336	0.6958	**0.9669**
6	0.030	0.4018	**0.8710**	0.00526	0.7194	**0.9696**
7	0.118	0.4040	**0.8502**	0.00678	0.7188	**0.9699**
8	0.185	0.4067	**0.7566**	0.00797	0.7212	**0.9710**
9	0.239	0.4076	**0.7181**	0.00939	0.7458	**0.9756**
10	0.284	0.4093	**0.7070**	0.01154	0.7683	**0.9769**

4. Summary & Conclusions

In most of the quality control applications, the data is assumed to follow normal distribution and the Shewart constants are used rigorously . These tables show that for a true SBLD if the Shewart limits are used in a mechanical way it would result in less confidence coefficient about the decision of process variation for mean,median,midrange, range and standard deviation charts . Hence if a data is confirmed to follow SBLD, the usage of Shewart constants in all the above charts is not advisable and exclusive evaluation and application of SBLD constants is preferable in statistical quality control.

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