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Selective Maintenance & Redundancy Allocation Problem with Interval Coefficients

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Abstract. This article is an attempt to obtain a compromise allocation of a reliability optimization problem in case of maintenance components and in case of redundancy. In first case, time taken and the cost spent on system maintenance is minimized simultaneously for the required reliability $R*$ (say) and in case second cost, weight and volume of the system is minimized simultaneously for the required reliability $R*$ (say). Since in real world problems decision makers do not know the precise value of parameters like time, cost, weight, volume etc, therefore, to deal with these uncertainties we assume these parameters as interval numbers. The problem of obtaining compromise allocation is formulated as Bi-Objective Selective Maintenance Allocation Problem (BSMAP) and Redundancy Allocation Problem (RAP) with interval coefficients in the objective functions and solved by goal programming technique with separation method. A numerical example is also presented to illustrate the computational details.

1. Introduction

In most real-world situations, the coefficients or input parameters of the model are not exactly known because the relevant data is scarce, the system is subject to changes etc, that is, the input parameters are uncertain in nature. In these circumstances Interval programming (IP) is one of the techniques to tackle these uncertainties in the constraints as well as in the objective functions or in both. The IP can be transformed into two sub-models, which correspond to worst lower bound and best upper bounds of desired objective function value. For this we develop methods that find the best optimum and worst optimum and the coefficient settings which achieve these two extremes. Moore (1966) was the first who introduce Interval analysis. After that many authors work in the field of interval linear programming such as Ishibuchi and Tanaka (1990) formulate a multi-objective programming in optimization with interval objective function, Inuiguchi and Sakawa (1995) obtain minimax regret solution to linear programming problems with an interval objective function, Chanas and Kuchta (1996) proposed a generalized approach for multi-objective programming in optimization of interval objective function, Chinneck and Ramadan (2000) solve linear programming with interval coefficients, Sengupta *et al.* (2001) done interpretation of inequality constraints involving interval coefficients and obtain a solution to interval linear programming, Pandian and Natarajan (2010) proposed a new method for finding an optimal solution of fully interval integer transportation problems, Huang and Cao (2011) analyze the solution methods for interval linear programming, and several other authors developed different procedures to deal with these problems.

Keywords. Selective Maintenance; Goal programming; Separation method; Interval programming; Redundancy; Compromise allocation

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In the production of goods almost every industry use systems and if such systems deteriorate or fail, then effect can be wide spread. This deterioration is often resulting in higher production cost, time, lower product quality and quantity. The decision of system maintenance is taken on the basis of the state condition of the system (*i.e.* whether the system is good or bad). Increasingly the engineers are employing optimization as a system maintenance tool for finding optimal maintenance action characterized by lower cost while satisfying performance requirements.

A maintained system is a system in which the failed or deteriorated components can be made maintained so as to operate normally. Rice *et al.* (1998) were the first who spread light in this area and after that many more authors discuss and formulate the selective maintenance allocation problem for a series – parallel system as mathematical programming problem. Some of the recent work in this area are due to Ali *et al.* (2011a, 2011b, 2011c, 2012, 2013) and Khan and Ali (2012), Ali and Hasan (2012), Gupta *et al.* (2013) and many other authors.

There exist several methods to improve the system reliability e.g. maintenance of the system, increasing the reliability of components through a product improvement program, using structural redundancy etc. A good deal of effort has been centered in the field of redundancy allocation problem (RAP). The RAP is a well-known system reliability optimization problem which can be formulated as a difficult nonlinear integer program. It has been extensively studied and solved using many different mathematical programming and heuristic approaches. Some of the work in this direction are due to Coit and Smith (1995), Coit and Smith (1996), Misra (1971), Kuo and Prasad (2000), Kulturel-Konak *et al.* (2003), Ramirez-Marquez and Coit (2004), Yalaoui *et al.* (2005) and many others.

In this article, BSMAP and RAP with interval coefficients in objective functions are formulated and the compromise allocation are obtained by an algorithm developed on combining goal programming technique and separation method. Rest of the article is organized as follows: In section 2 some interval arithmetic operations are discussed while in section 3 interval linear programming is defined. Section 4 is devoted to the problem formulation and section 5 to solution procedure. In section 6 a numerical example is given to demonstrate the computational details and finally section 7 conclude the work.

2. Interval arithmetic

The number whose exact value is not known is an interval number, but a range of possibilities is known (Moore et al., 2009). Interval number consist of both lower and upper bounds, $\tilde{x} \in [\underline{x}, \overline{x}]$, where $\underline{x} \leq \overline{x}$.

Interval arithmetic defines a set of operations on intervals. Let $\tilde{x}_1 = [\underline{x}_1, \overline{x}_1]$ *and* $\tilde{x}_2 = [\underline{x}_2, \overline{x}_2]$be two interval numbers. The following operations can be defined (Moore et al., 2009):

$$\begin{aligned}
&\tilde{x}_1 + \tilde{x}_2 = [\underline{x}_1 + \underline{x}_2,\, \overline{x_1} + \overline{x_2}] \\
&\tilde{x}_1 - \tilde{x}_2 = [\underline{x}_1 - \overline{x_2},\, \overline{x_1} - \underline{x}_2] \\
&\tilde{x}_1 \times \tilde{x}_2 = [\min(\underline{x}_1\underline{x}_2,\, \underline{x}_1\overline{x_2},\, \overline{x_1}\underline{x}_2,\, \overline{x_1 x_2}),\ \max(\underline{x}_1\underline{x}_2,\, \underline{x}_1\overline{x_2},\, \overline{x_1}\underline{x}_2,\, \overline{x_1 x_2})] \\
&\tilde{x}_1 \div \tilde{x}_2 = [\underline{x}_1 \times \overline{x_1}] \times \left[\frac{1}{\overline{x_2}}, \frac{1}{\underline{x}_2}\right]
\end{aligned}$$

When $\tilde{x} \in [\underline{x}, \overline{x}]$ is an interval number, its absolute value is the maximum of the absolute value of its endpoints: $|\tilde{x}| = \max\left(|\underline{x}|, |\overline{x}|\right)$ (Huang, 1994).

The center,x_c and width,x_w of a grey number $\tilde{x} \in [\underline{x}, \overline{x}]$ is defined as follows:

$$\begin{aligned}
x_c &= \tfrac{1}{2}(\underline{x} + \overline{x}) \\
x_w &= \tfrac{1}{2}(\overline{x} - \underline{x})
\end{aligned}$$

It is easily verifiable that$\overline{x} = x_c + x_w$ and $\underline{x} = x_c - x_w$.

3. Interval Programming

A mathematical programming problem whose coefficients are interval numbers is known as Interval programming problem. The standard form of Interval programming problem can be formulated as follows:

$$\begin{aligned} &\max\,(\min)\, Z = \textstyle\sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \\ &\textit{subject to} \ \ \textstyle\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i, \ \ i = 1, 2, \ldots, m \\ &\qquad\qquad \tilde{x}_j \geq 0, \quad j = 1, 2, \ldots, n \end{aligned}$$

where

$\tilde{x}_j \in [\underline{x}_j, \overline{x}_j]$; $j = 1, 2, \ldots, n$ are the interval decision variables.

$\tilde{c}_j \in [\underline{c}_j, \overline{c}_j]$; $j = 1, 2, \ldots, n$ are the interval objective functions coefficients.

$\tilde{a}_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$; $j = 1, 2, \ldots, n$, $i = 1, 2, \ldots, m$; $\tilde{b}_i \in [\underline{b}_i, \overline{b}_i]$, $i = 1, 2, \ldots, m$ are the interval constraints coefficients and right hand sides.

4. Formulation of the problem

Consider a system which is a series arrangement of m subsystems and performing a sequence of identical production runs.

Suppose that after completion of a particular production run, each component in the system is either functioning or failed. Ideally all the failed components in the subsystems are repaired and then replaced back prior to the beginning of the next production run. However, due to constraints on time and cost, it may not be possible to repair all the failed components in the system. In such situation, a method is needed to decide which failed components should be repaired and replaced back prior to the next production run and the rest be left in a failed condition. This process is referred to as selective maintenance (See Rice et al. 1998). In the selective maintenance the number of components available for the next production run in the i^{th} subsystem will be

$$(n_i - a_i) + d_i, \quad i = 1, 2, \ldots, m \tag{1}$$

<u>Case I: Interval programming in case of maintenance components</u>

The reliability of the given system is defined as

$$R = \prod_{i=1}^{m} \{1 - (1 - r_i)^{n_i - a_i + d_i}\} \tag{2}$$

The maintenance time in interval number for the system is given as

$$\sum_{i=1}^{m} [t_i^l, \, t_i^u] \ [d_i + \exp(\theta_i d_i)] \tag{3}$$

where $[t_i^l, \, t_i^u]$ is the time required to repair a component in i^{th} subsystem and $\exp(\theta_i a_i)$ is the additional time spent due to the interconnection between parallel components (Wang *et al.* (2009)).

The maintenance cost in interval number for the system is defined as

$$\sum_{i=1}^{m} [c_i^l, \, c_i^u] \ [d_i + \exp(\beta_i d_i)] \tag{4}$$

where $\exp(\beta_i d_i)$ is the additional cost spent due to the interconnection between parallel components (Wang *et al.* (2009)).

Therefore, the problem is formulated to minimize the time taken and the cost spent on system maintenance as different objectives for the required reliability R^* (say). The mathematical model of Interval Bi-Objective Selective Maintenance Allocation Problem (IBSMAP) is as follows:

$$\left.\begin{array}{l} Min\, C = \sum_{i=1}^{m}[c_i^l, c_i^u]\,\{d_i + \exp(\beta_i d_i)\} \\ and \\ Min\, T = \sum_{i=1}^{m}[t_i^l, t_i^u]\,\{d_i + \exp(\theta_i d_i)\} \\ \text{subject to} \\ \prod_{i=1}^{m} 1-(1-r_i)^{n_i-a_i+d_i} \geq R^* \\ \text{and} \quad 0 \leq d_i \leq a_i\,,\ i=1,2,...,\ m \text{ and integers} \end{array}\right\} \tag{5}$$

Case II: Interval programming in case of redundancy

To make manufacturing systems or product components to be more competitive in the market reliability should be improved and typical approaches to achieve higher reliability are:

- Increasing the reliability of system components, and
- Using redundant components in various subsystems in the system.

In this case, a reliability redundancy allocation problem is formulated to minimizing cost, weight and volume of the system as different objectives for a required reliability as follows:

$$\left.\begin{array}{l} Min\, C = \sum_{i=1}^{m}[c_i^l, c_i^u]\,\{n_i + \exp(\beta_i n_i)\} \\ Min\, W = \sum_{i=1}^{m}[w_i^l, w_i^u]n_i \\ and \\ Min\, V = \sum_{i=1}^{m}[v_i^l, v_i^u]n_i^2 \\ \text{subject to} \\ \prod_{i=1}^{m}\{1-(1-r_i)^{n_i}\} \geq R^* \\ \text{and} \quad n_i \geq 1\,,\ i=1,2,...,\ m \text{ and integers} \end{array}\right\}$$

5. Goal programming approach with Separation method

Solution Procedure for case I

In this section Interval Bi-Objective Selective Maintenance Allocation Problem is solved by goal programming approach but firstly we have to find upper bound (worst) and lower bound (best) for corresponding objective functions. Since the cost and time coefficients in the objective functions are interval valued, to obtain the upper and lower bounds we use separation method which proceeds as follows:

Step 1: Construct upper bound BSMAP of the given IBSMAP.

Step 2: Solve the upper bound BSMAP by the optimizing software LINGO. Let d_i^* for all i be an optimal solution of the upper bound BSMAP.

Step 3: Construct lower bound BSMAP of the given IBSMAP.

Step 4: Solve the lower bound BSMAP with the upper bound constraints $d_i^{'} \leq d_i^*$ for all i by the optimizing software LINGO. Let $d_i^{'*}$ be an optimal solution of the lower bound BSMAP with $d_i^{'*} \leq d_i^*$ for all i.

Step 5: $d_i^*,\ d_i^{'}$ provides the required upper and lower bounds of the given IBSMAP.

Now to formulate the goal programming model of the problem, the objectives in (5) are to be transformed into conventional goals in GP and introducing under and over deviational variables to each of them.

Then the equivalent goal expressions are:

$$\sum_{i=1}^{m} c_i^U\{d_i + \exp(\beta_i d_i)\} + d_{Lk}^- - d_{Lk}^+ = C^l$$

$$\sum_{i=1}^{m} c_i^L\{d_i + \exp(\beta_i d_i)\} + d_{Uk}^- - d_{Uk}^+ = C^u$$

Table 1: The parameters for the numerical example

Subsytems	n_i	r_i	θ_i	β_i	c_i	a_i	t_i	v_i	w_i
1	10	0.55	0.25	0.25	[6,10]	7	[2,4]	[1,2]	[3,6]
2	8	0.45	0.25	0.25	[5,9]	5	[3,5]	[2,3]	[2,4]
3	12	0.50	0.25	0.25	[5,11]	8	[2,4]	[2,3]	[4,7]

$$\sum_{i=1}^{m} t_i^U\{d_i + \exp(\beta_i d_i)\} + d_{Lk}^- - d_{Lk}^+ = T^l$$

$$\sum_{i=1}^{m} t_i^L\{d_i + \exp(\beta_i d_i)\} + d_{Uk}^- - d_{Uk}^+ = T^u$$

where (d_{Lk}^-, d_{Uk}^-) and $(d_{Lk}^+, d_{Uk}^+) \geq 0$ with $d_{Lk}^-.d_{Lk}^+ = 0$ & $d_{Uk}^-.d_{Uk}^+ = 0,\ k = 1, 2, ..., K$ represent the under and over deviational variables respectively and they are associated with the respective k-th goal. And $[C^l, C^u]$, $[T^l, T^u]$ are the upper and lower bounds obtained by separation method. Objective function of GP model is constructed by minimizing the unwanted deviational variables of the goals. Therefore, the executable GP model of the problem appears as:

Find $d_i,\ i = 1, 2, ..., m$ so as to

$$\left.\begin{array}{l} Minimize\ \sum_{k=1}^{K}(d_{Lk}^- + d_{Uk}^+) \\ subject\ to \\ \sum_{i=1}^{m} c_i^U\{d_i + \exp(\beta_i d_i)\} + d_{Lk}^- - d_{Lk}^+ = C^l \\ \sum_{i=1}^{m} c_i^L\{d_i + \exp(\beta_i d_i)\} + d_{Uk}^- - d_{Uk}^+ = C^u \\ \sum_{i=1}^{m} t_i^U\{d_i + \exp(\beta_i d_i)\} + d_{Lk}^- - d_{Lk}^+ = T^l \\ \sum_{i=1}^{m} t_i^L\{d_i + \exp(\beta_i d_i)\} + d_{Uk}^- - d_{Uk}^+ = T^u \\ \prod_{i=1}^{m} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq R^* \\ \text{and} \quad 0 \leq d_i \leq a_i,\ i = 1, 2, ...,\ m \text{ and integers} \end{array}\right\} \tag{6}$$

Similarly goal programming model is formulated for case II and solved by optimizing software LINGO to obtain the compromise allocation.

6. A Numerical Example

To demonstrate the computational details of the formulated problems and algorithm we consider a hypothetical system consisting of 3 subsystems. For the system the desired reliability requirement is $R^* \geq 0.97$. The other parameters for the various subsystems are given in table 1.

Following the steps given in section 3, the required upper and lower bounds are given as follows:

$$\left.\begin{array}{l} Minimize\ 10(d_1 + e^{0.25d_1}) + 9(d_2 + e^{0.25d_2}) + 11(d_3 + e^{0.25d_3}) \\ subject\ to \\ \prod_{i=1}^{3} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq 0.97 \\ 0 \leq d_1 \leq 7 \\ 0 \leq d_2 \leq 5 \\ 0 \leq d_3 \leq 8 \ \text{ and integers} \end{array}\right\} \tag{7}$$

Solving the above problem (7) by LINGO software, we get the upper bound for first objective function i.e. $d_1^* = 3,\ d_2^* = 5,\ d_3^* = 3$ with $C^u = 99.90554$

For lower bound we have to solve the following problem

$$\left.\begin{array}{l} Minimize\ 6(d_1+e^{0.25d_1})+5(d_2+e^{0.25d_2})+5(d_3+e^{0.25d_3}) \\ subject\ to \\ \prod_{i=1}^{3} 1-(1-r_i)^{n_i-a_i+d_i} \geq 0.97 \\ 0 \leq d_1 \leq 3 \\ 0 \leq d_2 \leq 5 \\ 0 \leq d_3 \leq 3 \text{ and integers} \end{array}\right\} \quad (8)$$

After solving (8) by LINGO, we get $d_1^* = 3,\ d_2^* = 5,\ d_3^* = 3$ with $C^l = 53.71900$.
Similarly we get the upper and lower bound for the second objective which is given below:

$T^l = 24.48937$ and $T^u = 44.88968$.

Using the above bounds now we formulate the GP model as:

$$\left.\begin{array}{l} Minimize\ \sum_{k=1}^{2}(d_{Lk}^- + d_{Uk}^+) \\ subject\ to \\ 10(d_1+e^{0.25d_1})+9(d_2+e^{0.25d_2})+11(d_3+e^{0.25d_3})+d_{L1}^- - d_{L1}^+ = 53.71900 \\ 6(d_1+e^{0.25d_1})+5(d_2+e^{0.25d_2})+5(d_3+e^{0.25d_3})+d_{U1}^- - d_{U1}^+ = 99.90554 \\ 4(d_1+e^{0.25d_1})+5(d_2+e^{0.25d_2})+4(d_3+e^{0.25d_3})+d_{L2}^- - d_{L2}^+ = 24.48937 \\ 2(d_1+e^{0.25d_1})+3(d_2+e^{0.25d_2})+2(d_3+e^{0.25d_3})+d_{U2}^- - d_{U2}^+ = 44.88968 \\ \prod_{i=1}^{3} 1-(1-r_i)^{n_i-a_i+d_i} \geq 0.97 \\ 0 \leq d_1 \leq 7 \\ 0 \leq d_2 \leq 5 \\ 0 \leq d_3 \leq 8 \text{ and integers} \end{array}\right\} \quad (9)$$

The above problem (9) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get the compromise solution as $d_1^* = 4,\ d_2^* = 4,\ d_3^* = 2$ also, the minimum cost and time are [92.1448, 167.7834], [40.8889, 75.0595].
Using the procedure discussed in section 5, we can formulate the Goal programming model for case II as:

$$\left.\begin{array}{l} Minimize\ \sum_{k=1}^{3}(d_{Lk}^- + d_{Uk}^+) \\ subject\ to \\ (10(n_1+e^{0.25n_1})+9(n_2+e^{0.25n_2})+11(n_3+e^{0.25n_3}))+d_{L1}^- - d_{L1}^+ = 111 \\ (6(n_1+e^{0.25n_1})+5(n_2+e^{0.25n_2})+5(n_3+e^{0.25n_3}))+d_{U1}^- - d_{U1}^+ = 207 \\ (6n_1e^{0.25n_1}+4n_2e^{0.25n_2}+7n_3e^{0.25n_3})+d_{L2}^- - d_{L2}^+ = 60 \\ (3n_1e^{0.25n_1}+2n_2e^{0.25n_2}+4n_3e^{0.25n_3})+d_{U2}^- - d_{U2}^+ = 113 \\ (2n_1^2+3n_2^2+3n_3^2)+d_{L3}^- - d_{L3}^+ = 245 \\ (1n_1^2+2n_2^2+2n_3^2)+d_{U3}^- - d_{U3}^+ = 392 \\ \prod_{i=1}^{3} 1-(1-r_i)^{n_i} \geq 0.97 \\ n_i \geq 1,\ i=1,\ldots,3 \text{ and integers} \end{array}\right\} \quad (10)$$

The above problem (10) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get the compromise solution as $n_1^* = 7,\ n_2^* = 7,\ n_3^* = 7$ also, the minimum cost, weight and volume are [116.43, 218.31], [63, 119], [245, 392].

7. Conclusion

In practical situations uncertainty is an unavoidable characteristic of mathematical modeling. Often, the one don't have enough information to determine the exact values of the model and the parameters have to be estimated. Interval numbers express the system's information in a range and provide a great flexibility in modeling the uncertain systems. In this article, we consider a BSMAP and a RAP with interval coefficients in the objective functions and solve the formulated problems by goal programming approach. To obtain the GP model we need upper and lower bounds of the individual objective functions for this purpose Separation method is used and a combined algorithm of goal programming and separation method is given in section 5. Finally the GP model is solved by optimizing software LINGO to obtain the compromise allocation. In the end a numerical example is given to demonstrate the algorithm.

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