ProbStat Forum, Volume 09, January 2016, Pages 50–56
ISSN 0974-3235

ProbStat Forum is an e-journal.
For details please visit www.probstat.org.in

Annualized hours planning with fuzzy demand constraint

Md. Gulzar ull Hasan, Irfan Ali, S S Hasan

Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh-202 002, India.

Abstract.

In this paper we have considered the problem of Annualized hours (AH). AH is a method of distributing working hours with respect to the demand over a year. The AH problem is formulated as a mathematical programming problem with fuzzy demand constraint to obtain an optimum allocation and total overtime cost. Finally, An illustration is given with a computational experiment to demonstrate and analyse the effectiveness of the model.

1. Introduction

The workforce management is an important aspect for every industry. One of the major reason of it's need is the variation in demand of skilled workers and variation in their availability throughout the year. In different countries people are employed on different basis like on monthly salary, five day week salary, number of jobs, number of working hours etc. In Countries like Germany, United Kingdom, Switzerland people are hired on annual contract of working hours. Annualized hours policy helps organization to efficiently match workforce demand and availability. As it is very helpful in labor intensive industries like health care.

Many author worked with deterministic demand like Hung[1999a,b], Widmer[2004], Azmat *et al.*[2004], E. van der Veen *et al.*[2014] but Lusa *et al.*[2008] worked with stochastic demand in annualized hours problem.

In this paper, we have modeled a cost effective allocation of workers with uncertainty in form of fuzziness involved in demand constraint. The workers are hired on an annual contract for number of working hours. It gives an advantage that salary will be given only for the work done. The contract gives a security for the availability of worker throughout the year to the employer and also security to the employee for the availability of the job.

The annual contract for number of working hours enables the employer to let the employee work for more hours in some period and less in others. The annualized hours problem, calculates the optimal distribution of available hours capacity throughout the year. In addition to annualized hours, we consider a multi tasking staffing problem.

2010 *Mathematics Subject Classification.*
Keywords. Annualized Hours, Fuzzy Demand, Mathematical Programming, Optimum Allocation.
Received: 07 April 2015; Accepted: 06 November 2015

Email address: irfii.st@amu.ac.in (Irfan Ali)

2. Fuzzy integer linear programming(FILP)

Zimmermann[1991] categorized uncertainty as stochastic uncertainty and fuzziness. Fuzziness appears when the information is vague, imprecise or ambiguous or when the information is not clearly defined. The concept of fuzzy set theory is first introduced by Zadeh[1965]. In 1970 Bellmann and Zadeh[1970] developed a decision theory in fuzzy environment. Here we are interested in Fuzzy integer linear programming(FILP). An Integer linear programming problem with fuzziness involved in it is known as fuzzy integer linear programming.

The fuzziness can appear in different forms i.e. with fuzzy inequalities, fuzzy objective function, both fuzzy inequalities and fuzzy objective function and fuzzy parameters. An Integer programming problem with fuzzy inequalities can be given as follows:

$$\text{Max } z = cx \tag{1}$$

$$\sum_{j\in N} a_{ij}x_j \lesssim b_i, i \in M = (1, 2, \dots, m) \tag{2}$$

$$x_j \geq 0, j \in N = (1, 2, \dots, n) \tag{3}$$

$$x_j \in N \tag{4}$$

In the above problem the constraints are fuzzy, it means that the decision maker is willing to permit some violations in the accomplishment of the constraints. Herrera and Verdegay[1995] showed that the above problem is equivalent to a crisp parametric Integer linear programming problem(ILPP). As, Fuzziness appeared in constraints we need to define a membership function to disappear fuzziness.

$$\mu_i : R^n \to (0, 1], i \in M. \tag{5}$$

each of which gives the degree to which each $x \in R^n$ accomplishes the respective constraint.

If $a_ix \leq b_i$ then the i^{th} constraint is absolutely satisfied, where as if $a_ix \geq b_i + d_i$, where d_i is the maximum tolerance from b_i, as determined by the decision maker, then the i^{th} constraint is absolutely violated. For $a_ix \in \{b_i, b_i + d_i\}$ the membership function is monotonically decreasing.

For the sake of convenience we consider a linear membership function for the above problem. Therefore, the linear membership function for i^{th} fuzzy constraint can be defined as,

$$\mu_i(x) = \begin{cases} 1 & , \text{ if } a_ix \leq b_i, \\ \dfrac{[(b_i + d_i) - a_ix]}{d_i} & , \text{ if } b_i \leq a_ix \leq b_i + d_i \\ 0 & , \text{ if } a_ix \geq b_i + d_i \end{cases} \tag{6}$$

Now using membership function define in (6) following auxiliary parametric ILP from (1)-(4) will be obtained:

$$\text{Max } z = cx \tag{7}$$

$$\sum_{j\in N} a_{ij}x_j \leq b_i + d_i(1 - \alpha), i \in M = (1, 2, \dots, m) \tag{8}$$

$$x_j \geq 0, j \in N = (1, 2, \dots, n) \tag{9}$$

$$x_j \in N, \quad \alpha \in (0, 1] \tag{10}$$

In (7) the cost associated with the decision variable is a crisp number.

3. Problem formulation

The problem studied and modeled in this paper is based upon the minimization of total overtime cost expenditure on workers and also determining the weekly overtime working hours by each worker.
The characteristics of the problem are summarized below:
1) Planning period of the allocation of workforce is taken as 26 weeks(approx. 6 months)
2) No holiday week is considered in this problem.
3) Workers are employed on an annual contract of fixed number of working hours for each week.
4) Different types of tasks are taken with the assumption that the staff is multitasking.
4) Overtime is permitted only by 20 percent of the contracted hours of each worker.
6) Hiring temporary workers is not possible.
7) A Utility function is to be optimised.

Here, we are defining sets, parameters and variables which will be used throughout the paper. The list of notations is given below:

Sets, parameters

I	set of Workers indexed by i
J	set of Tasks indexed by j
T	set of Week indexed by t
d_{jt}	demand for Task t
C_i	fixed cost of workers i for an annual contract
L_{it}, U_{it}	minimum, maximum number of Working hours contracted by worker i in time slot t
L_i, U_i	minimum, maximum number of Working hours contracted by worker i in entire planning horizon T
p_{jt}	maximum tolerance from d_{jt}
α	membership grade of p_{jt}

Variables

S_{ijt}	number of working hours alloted to worker i on task j during week t
S_{it}	number of working hours alloted to worker i during week t i.e. $S_{it} = \sum_{j \in J} S_{ijt}$
Y_i	1 if surgeon i is selected in the workforce, 0 if not
TC	total annualized cost of workers for both the tasks

The mathematical programming model of above problem under fuzzy constraints is as follows:

The total number of hours alloted to workers should be greater than the demand for each task and in each week

$$\sum_{i \in I} S_{ijt} \gtrsim d_{jt}, j \in J, t \in T \tag{11}$$

Define an auxiliary variable S_{it} as the sum of total hours alloted in each task for each worker and week as follows:

$$S_{it} = \sum_{j \in J} S_{ijt}, i \in I, t \in T \tag{12}$$

The constraint shows that number of working hours alloted to each worker in every week is between the lower and the upper bound:

$$L_{it} Y_i \leq S_{it} \leq U_{it} Y_i, i \in I, t \in T \tag{13}$$

The next constraint shows that number of working hours alloted to each worker in planning horizon i.e. about 26 weeks is between the lower and the upper bound:

$$L_i Y_i \leq \sum_{t \in T} S_{it} \leq U_i Y_i, i \in I \tag{14}$$

The objective function is given as:

$$\text{Min Total cost(TC)} \ = \sum_{i \in I} C_i(\sum_{t \in T} S_{it} - L_i Y_i) \tag{15}$$

Now applying the non negativity and integer restrictions on the variables:

$$S_{ijt} \geq 0 \text{ and an Integer } , i \in I, j \in J, t \in T \tag{16}$$

and

$$Y_i \in \{0, 1\}, i \in I \tag{17}$$

Now, By Herrera and Verdegay[1995] the Crisp formulation of the above mathematical programming model from (11)-(17) will be as follows:

$$\text{Min Total cost(TC)} = \sum_{i \in I} C_i(\sum_{t \in T} S_{it} - L_i Y_i)$$

$$\sum_{i \in I} S_{ijt} \geq d_{jt} + (1 - \alpha) p_{jt}, j \in J, t \in T$$

$$S_{it} = \sum_{j \in J} S_{ijt}, i \in I, t \in T$$

$$L_{it} Y_i \leq S_{it} \leq U_{it} Y_i, i \in I, t \in T$$

$$L_i Y_i \leq \sum_{t \in T} S_{it} \leq (U_i) Y_i, i \in I$$

$$S_{ijt} \geq 0 \text{ and an Integer } , i \in I, j \in J, t \in T$$

$$Y_i \in \{0, 1\}, i \in I$$

and

$$\alpha \in [0, 1]$$

4. Experimental study

For the above model we have taken a experimental case problem. Suppose that 3 Workers are working in a small industry with specialisation in 2 types of tasks. These workers are hired on an annual contract of fixed number of Working hours. The planning period is taken as 26 weeks(about 6 months).

The basic data used for the experiment are as follows:

1) No. of Workers $I = 1, 2, 3$.
2) No. of Tasks $J = 1, 2$.
3) No. of Weeks $T = 1, ..., 26$.
4) L_{it}= 35 hours and U_{it}= 50 hours.

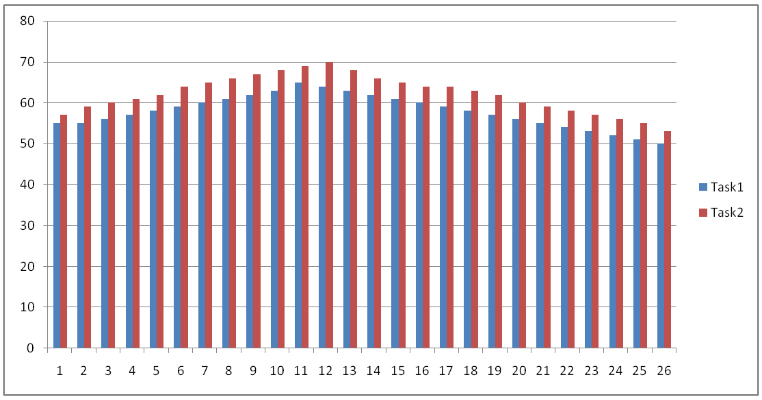

Figure 1: Demand pattern of task 1 and task 2

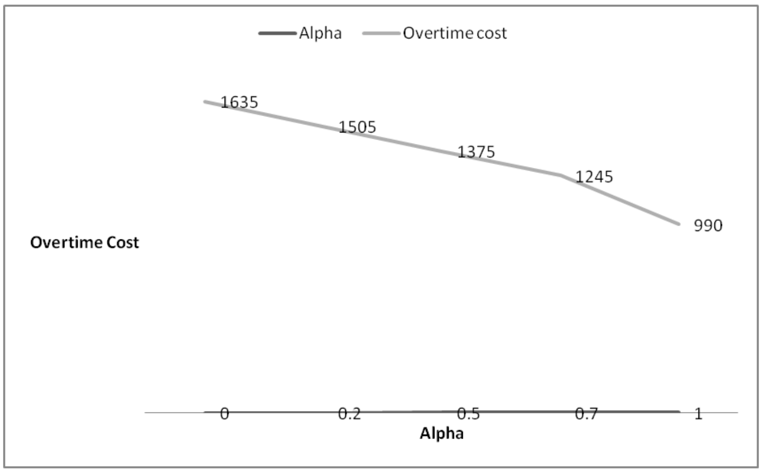

Figure 2: Total Cost trend

5) L_i= 910 hours and U_i= 1100 hours.
6) Cost of each worker for each hour is 2.5 Rs(for calculation simplification purpose).
7) Tolerance level for the demand constraint is p_{jt}= 5.
The pattern of demand for the tasks 1 and 2 is:

Now, we want to obtain the number of shortage hours for each worker in every week of planning period under fuzzy demand constraints by optimizing the utility function i.e. minimizing the cost of overtime.

5. Case Results

On solving the proposed mathematical programming model with 263 variables and 368 constraints for the above data using LINGO 13.0, we obtain the following results:
The Total overtime cost with respect to the membership grade α is decreasing. As α increases the total cost decreases. The trend of cost with respect to α is shown graphically in figure 2.

Overtime by each worker in each week is also obtained and shown graphically in figure 3 and 4. The overtime hours by each worker for values of $\alpha = 0$ and 1 is presented in table 1. It is observed that More Overtime hours are needed for lower values of α while less overtime hours are needed for higher value of α.

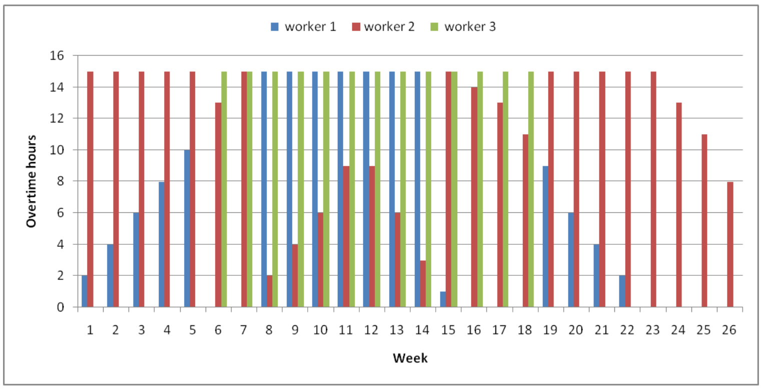

Figure 3: Overtime for $\alpha = 0$

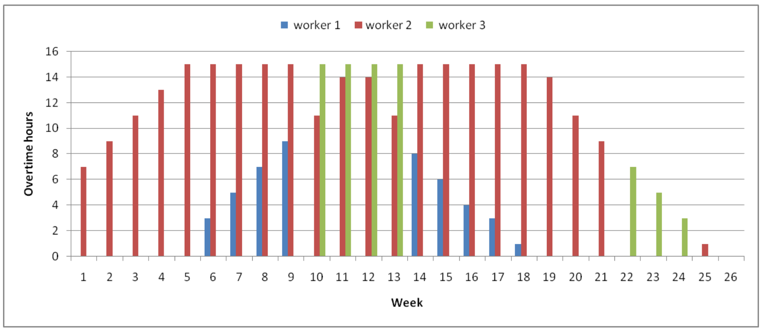

Figure 4: Overtime for $\alpha = 1$

α	Worker 1	Worker 2	Worker 3
0	46	275	75
1	157	302	195

Table-1

The fuzzy approach helps the Decision maker to prepare the budget for the hiring cost of workers with respect to the satisfaction level of the demand constraint. It gives choice to the Decision maker.

6. Conclusion

In this paper we modeled a cost effective allocation problem of workers. Workers are hired on an annualized hours contract. We modeled this problem as a mathematical programming problem. As per our knowledge our paper contributes first work in cost effective allocation of workers with multi-specialization and annualized hours contract, when the demand constraint is uncertain in form of Fuzziness.
Also an hypothetical experimental study of mathematical model has been done to show effectiveness of the model. The results helps the employer to prepare their budget for the hiring cost of workers with respect to the membership grade of tolerance level of demand i.e. p_{jt}. The overtime for each worker is also obtained.

7. Acknowledgement

The Second author is thankful to the University grants commission(India) for providing BSR startup grant(F.30-90/2015) to carry out this work efficiently.

References

[1] Azmat C.S. and Widmer M. (2004). A case study of single shift planning and scheduling under anhours: A simple three-step approach. *European Journal of Operational research*, 153(1):148-175.

[2] Azmat C.S., Hrlimann T. and Widmer M. (2004): Mixed integer programming to schedule a single-shift workforce under annaulized hours. *Annals of Operations research*, 128(1):199-215.

[3] Herrera. F and Verdegay J.L. (1995): Three models of fuzzy integer linear programming. *European Journal of Operational research*, 83:581-593.

[4] H. J. Zimmermann(1991), *Fuzzy Set Theory and Its Applications*(2nd Ed.), Kluwer-Nijhoff, Hinghum.

[5] Hung R (1999a): A multiple-shift workforce scheduling model under annualized hours.*Nav Res Logist*, 46(6):726-736.

[6] Hung R (1999b): Scheduling a workforce under annualized hours.*Int J Prod Res*, 37(11):2419-2427.

[7] L. A. Zadeh, Fuzzy sets, *Information and Control*, 1965, 8: 338-353.

[8] Lusa A., Corominas A., Munoz N. (2008): A multistage scenario optimisation procedure to plan annualised working hours under demand uncertainty. *Int J Prod Econ*, 113(2):957-968.

[9] R. E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, *Management Science*, 1970, 17: B-141-164.

[10] Van der Veen E., Erwin W. Hans, Bart Veltman, Leo M. Berrevoets, Hubert J.J.M. Berden(2014): A case study of cost-efficient staffing under annualized hours. *Health Care Manag. Sci.*.