

A Group Acceptance Sampling Plans Based on Truncated Life Tests for Type-II Generalized Log-Logistic Distribution

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Abstract. In this paper, we have developed a group acceptance sampling plan for truncated life tests when the lifetime of items follows the Type-II generalized log logistic distribution (TGLLD). At a specified level of consumer risk and at the predefined test termination time, derived the minimum number of groups required for a given group size and the acceptance number. The values of operating characteristic (OC) function according to various quality levels are derived and the minimum ratios of true average life to the specified life at given producers risk are obtained. The results are illustrated with example.

1. Introduction

Statistical quality control refers to the user of statistical methods in monitoring and maintaining of the quality of products and services. One method, referred to as acceptance sampling, can be used when a decision must be made to accept or reject a group of parts or items based on the quality found in a sample. Acceptance Sampling is an important field of statistical quality control that popularized by Dodge and Roming and originally applied by the U.S. military to the testing of bullets during World War II. Acceptance sampling is a statistical method of inspecting a sample of goods and deciding whether to accept the entire lot based on the results.

The acceptance sampling plans for truncated life tests are frequently used to determine the sample size from a lot under consideration. For usual sampling plan, it is assume that only a single item is put in a tester. Testers accommodating multiple number of items at a time in order to save the time and cost. For this type of testing the number of items to be equipped in a tester is given by the specification. The acceptance sampling plan under this type of testers will be called group acceptance sampling plan (GASP). If GASP is used in conjunction with truncated life tests, it is called a GASP based on truncated life test assuming that the lifetime of product follows a certain probability distribution. For such a test the determination of sample size is equivalent to determine the number of groups.

Many authors have discussed acceptance sampling plans based on truncated life tests of single item groups for various distributions. Kantam and Rosaiah (1998), Kantam et al. (2001), Rosaiah et al. (2006), Aslam and Jun (2009), Rao (2009, 2010, 2011), Aslam and Jun (2009a), Aslam et al. (2011), Ramaswamy

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and Anburajan (2012) and Rao et al. (2014). The main aim of this study is to develop a GASP based on truncated life tests assuming the life time distribution of the products follows TGLLD. Rao et al. (2012, 2013) have discussed reliability test plans for TGLLD and also developed Economic reliability test plans for TGLLD. The rest of the paper is arranged in the following way, introduction of TGLLD is given in Section 2 and the proposed GASP is discussed in Section 3. Description of the proposed methodology with real life data is given in Section 4 and finally, some conclusions are established in Section 5.

2. Type-II Generalized Log Logistic Distribution

Log logistic distribution (LLD) has proven its importance in quality control. Different types of acceptance sampling plans are developed for LLD.

The cumulative distribution function (cdf) of the log-logistic distribution (LLD) is

$$F(t) = \frac{\left(\frac{t}{\sigma}\right)^\lambda}{\left[1 + \left(\frac{t}{\sigma}\right)^\lambda\right]}; t > 0, \sigma > 0, \lambda > 1 \tag{1}$$

Rosaiah et al. (2008) introduced a generalization of LLD and called as Type-II generalized log logistic distribution (TGLLD), its cumulative distribution function (cdf) is

$$F(t; \lambda, \sigma, \theta) = 1 - \left[1 + \left(\frac{t}{\sigma}\right)^\lambda\right]^{-\theta}; t > 0, \sigma, \theta > 0, \lambda > 1 \tag{2}$$

It can be seen that this is the failure probability function of series system of θ components, where life times of the components are independently and identically distributed with the log logistic distribution given by (1) as a common model. It may be noted that the distribution given in (2) is defined through the reliability oriented generalization of log-logistic distribution. In short we call this as the Type-II generalized log logistic distribution [Type-I generalized (exponentiated) log logistic distribution is dealt with by Rosaiah et al. (2006)]. The corresponding probability density function (pdf) is given by

$$f(T; \theta, \lambda, \sigma) = \frac{\lambda\theta}{\sigma} \frac{\left(\frac{t}{\sigma}\right)^{\lambda-1}}{\left[1 + \left(\frac{t}{\sigma}\right)^\lambda\right]^{\theta+1}}; t > 0, \sigma, \theta > 0, \lambda > 1 \tag{3}$$

where σ is the scale parameter, λ and θ are shape parameters and θ takes an integer values. The $100q^{th}$ percentile of the TGLLD is given by

$$t_q = \sigma \left[(1-p)^{\frac{-1}{\theta}} - 1 \right]^{\frac{1}{\lambda}} = \sigma \eta_q \tag{4}$$

where

$$\eta_q = \left[(1-p)^{\frac{-1}{\theta}} - 1 \right]^{\frac{1}{\lambda}}$$

Hence, for fixed values of $\theta = \theta_0$ and $\lambda = \lambda_0$, the quantile t_q in equation (4) is the function of scale parameter $\sigma = \sigma_0$ i.e., $t_q \geq t_{q_0} \iff \sigma \geq \sigma_0$

where

$$\sigma_0 = \left(\frac{t_{q_0}}{\eta_q} \right) \tag{5}$$

It may be noted that σ_0 depends on θ_0 and λ_0 to draw the acceptance sampling plans for TGLLD.

3. Design of the proposed Sampling Plan

We prefer to use the percentile point as the quality parameter instead of mean and it is denoted by t_q . According to the acceptance sampling plans the hypothesis $H_0: t_q \geq t_{q_0}$ is tested based on the number of observed failures from a sample in a pre-fixed time. The number of products n in the lot to be tested and is divided into equal sized groups subject to the availability of the number of testers. It is assumed that the experimental time and number of products in each group are fixed in advance. Since $n = rg$ determining n is equivalent to determining g . The following group acceptance sampling plan on the truncated life test is proposed.

- i Select a random sample of n products and allocate r items to each of g groups so that $n = rg$
- ii Fix an acceptance number c and the pre-specified test termination time t_0
- iii Accept the lot if the number of failures from all groups is less than or equal to c .
- iv Terminate the experiment if more than c failures observed from all groups before the termination t_0 .

The probability of accepting lot for group sampling plan based on the number of failures from all groups under a truncated life test before the prefixed time t_0 is given by

$$F(p) = \binom{rg}{i} p^i (1 - p)^{rg-i} \tag{6}$$

where g is number of groups, c is the acceptance number, r is the group size and p is the probability of getting a failure within the test termination time t_0 . Since the lifetime of the product t follows TGLLD, we have $p = F(t_0)$. Since it is convenient to set the termination time as multiple of the targeted 100th lifetime percentile, t_{q_0} and a constant δ_q . Let t_q be the true 100th lifetime percentile. Then p can be rewritten as

$$p = 1 - \left[1 + \left(\frac{t_0}{\sigma} \right)^\lambda \right]^{-\theta} \tag{7}$$

In order to find the design parameters of the proposed group acceptance sampling plan, we prefer the approach based on two points on the curve by considering the producers and consumers risk. In this approach the quality level is measured through the ratio of its percentile lifetime to the true lifetime, $\frac{t_q}{t_{q_0}}$. To ensure and improve the quality of the products, producer may use the percentile ratios. Producer prefers that the probability of lot acceptance shall be at least $1 - \alpha$ at the acceptable reliability level (ARL), p_1 . Hence the producer wants that the lot to be accepted at different values of percentile ratios, say for the values of $\frac{t_q}{t_{q_0}} = 2, 4, 6, 8$ given in (7). In view of the consumer the lot rejection probability shall not be exceed β at the lot tolerance reliability level (LTRRL), p_2 . Hence, consumer may reject the lot if $\frac{t_q}{t_{q_0}} = 1$ according to the equation (7).

$$\sum_{i=0}^c \binom{rg}{i} p_1^i (1 - p_1)^{rg-i} \geq 1 - \alpha \tag{8}$$

$$\sum_{i=0}^c \binom{rg}{i} p_2^i (1 - p_2)^{rg-i} \leq \beta \tag{9}$$

where p_1 and p_2 are given by

$$p_1 = 1 - \left[1 + \left(\frac{\eta_q \delta_q^0}{\left(\frac{t_q}{t_{q_0}} \right)^\lambda} \right)^\lambda \right]^{-\theta} \quad \text{and} \quad p_2 = 1 - \left[1 + (\eta_q \delta_q^0)^\lambda \right]^{-\theta} \tag{10}$$

Estimates of the parameters of the proposed plan for different values of shape parameters are obtained. i.e., at the given values of $\lambda = 2, \theta = 2$, producers risk $\alpha = 0.05$ and test termination time $t_0 = \delta_q t_{q_0}$ with δ_q

= 0.5 or 1.0, the parameters of the proposed GASP are estimated for 10th and 50th percentiles at different confidence levels $\beta = 0.25, 0.10, 0.05, 0.01$. The plan parameters are presented in Tables 1 and 2 for $\lambda=2, \theta=2$ at 10th and 50th percentiles respectively. Tables 3 and 4 are constructed using ML estimates $\hat{\theta} = 1.8138$ and $\hat{\lambda} = 2.3381$ at 10th and 50th percentiles respectively. We noticed from Tables 1 to 4, as percentile ratio increases, the number of groups g decreases. Similarly, as r increases from 5 to 10, the number of groups reduces.

4. Description of the Proposed Methodology with Real Data Example

In this section, we use a real data set to show that the type-II generalized log logistic distribution can be a suitable model. Folks & Chhikara (1978) presented several sets of data to describe the Birnbaum-Saunders distribution. One of the data set gives the runoff amounts at Jug Bridge, Maryland. For ready reference this data set is reproduced as follows:

0.17, 0.23, 0.33, 0.39, 0.39, 0.40, 0.45, 0.52, 0.56, 0.59, 0.64, 0.66, 0.70, 0.76, 0.77, 0.78, 0.95, 0.97, 1.02, 1.12, 1.19, 1.24, 1.59, 1.74 and 2.92.

We show a rough indication of the goodness of fit for our model by plotting the superimposed for the data shows that the TGLLD is a good fit in Figure 1 and also goodness of fit is emphasized with Q-Q plot, displayed in Figure 1. The maximum likelihood estimates of the three parameter TGLLD for the runoff amounts are $\hat{\sigma} = 0.7616, \hat{\lambda} = 2.6602$ and $\hat{\theta} = 1.1772$ and using the Kolmogorov-Smirnov test, it is found that the maximum distance between the data and the fitted of the TGLLD is 0.0657 with p-value is 0.9999. Meanwhile, the maximum likelihood estimates of the two-parameter TGLLD for the runoff amounts are $\hat{\lambda} = 2.3381$ and $\hat{\theta} = 1.8138$ and the Kolmogorov-Smirnov test and found that the maximum distance between the data and the fitted of the TGLLD is 0.0867 with p-value is 0.9919. Therefore, the two-parameter TGLLD is also provides reasonable good fit for the runoff amounts.

Let us consider the life time of a product is known to follow a TGLLD with index parameters $\hat{\theta} = 1.8138$ and shape $\hat{\lambda} = 2.3381$. Suppose that it is desired to develop the group acceptance sampling plan to satisfy that the 50th percentile lifetime is greater than runoff amounts 0.35 through the experiment to be completed by runoff amounts 0.35 using tester taking with five products each. Let us fix that the consumer's risk is at 25% when the true 50th percentile is runoff amounts 0.35 and the producer's risk is 5% when the true 50th percentile is runoff amounts 0.70. Since $\hat{\theta} = 1.8138$ and $\hat{\lambda} = 2.3381$, the consumer's risk is 25%, $r = 5, \delta_q^0 = 0.5$ and $\frac{t_q}{t_{q0}} = 2$, the minimum number of groups and acceptance number given by $g = 2$ and $c = 3$ from Table 4. Thus the design can be implemented as follows: select a total of 10 products are needed and that two items will be allocated to each of the five testers. We can accept the lot when no more than three failures occurs before runoff amounts 0.35 from two groups. According to this plan, the runoff amounts could have been accepted because there are only three failure before the termination time, runoff amounts 0.35.

5. Conclusions

In this article, a group sampling plan to ensure the specified product lifetime percentile has been developed for the Type-II generalized log logistic distribution. The design parameters c and g of the proposed sampling plan are determined by the two-point method. Extensive tables have been provided for the industrial use according to various parameters and percentile values. It was observed that the number of groups required increases as the consumers confidence increases, true quality decreases and as r increases the number of groups reduces. A comparison between the proposed group sampling plan and the ordinary group sampling plan has also been discussed. It has been noticed that the proposed plan requires a smaller sample size than the ordinary group sampling plan does. The methodology illustrated with real data set.

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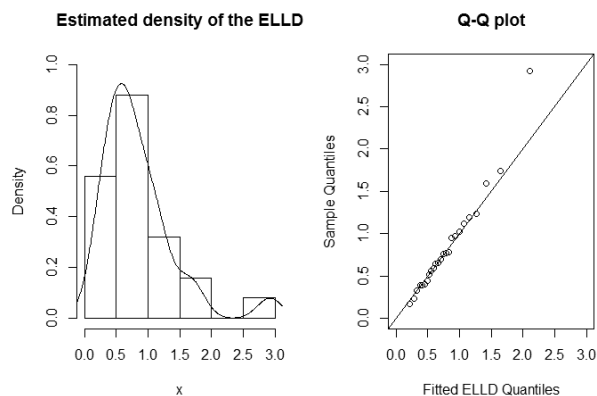


Figure 1: The density plot and Q-Q plot of the fitted type-II generalized log logistic distribution for the runoff amounts data

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Table 1: Proposed plan for TGLLD with $\theta=2$ and $\lambda=2$ for 10^{th} percentile

| β | $\frac{t_q}{t_0^q}$ | r=5 | | | | | | r=10 | | | | | |
|---------|---------------------|----------------|---|--------|----------------|---|--------|----------------|---|--------|----------------|---|--------|
| | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | |
| | | c | g | Pa | c | g | Pa | c | g | Pa | c | g | Pa |
| 0.25 | 2 | 23 | 5 | 0.9817 | - | - | - | 27 | 3 | 0.9708 | - | - | - |
| | 4 | 4 | 1 | 0.99 | 14 | 3 | 0.9736 | 7 | 1 | 0.9881 | 19 | 2 | 0.9922 |
| | 6 | 3 | 1 | 0.9912 | 8 | 2 | 0.9718 | 4 | 1 | 0.9566 | 8 | 1 | 0.9718 |
| 0.1 | 8 | 2 | 1 | 0.982 | 4 | 1 | 0.99 | 3 | 1 | 0.9686 | 7 | 1 | 0.9881 |
| | 2 | 27 | 6 | 0.9708 | - | - | - | 27 | 3 | 0.9708 | - | - | - |
| | 4 | 7 | 2 | 0.9881 | 14 | 3 | 0.9736 | 7 | 1 | 0.9881 | 19 | 2 | 0.9922 |
| 0.05 | 6 | 3 | 1 | 0.9912 | 8 | 2 | 0.9718 | 4 | 1 | 0.9566 | 8 | 1 | 0.9718 |
| | 8 | 2 | 1 | 0.982 | 4 | 1 | 0.99 | 3 | 1 | 0.9686 | 7 | 1 | 0.9881 |
| | 2 | 31 | 7 | 0.9606 | - | - | - | - | - | - | - | - | - |
| 0.01 | 4 | 7 | 2 | 0.9881 | 14 | 3 | 0.9736 | 7 | 1 | 0.9881 | 27 | 3 | 0.9708 |
| | 6 | 3 | 1 | 0.9912 | 8 | 2 | 0.9718 | 4 | 1 | 0.9566 | 8 | 1 | 0.9718 |
| | 8 | 2 | 1 | 0.982 | 4 | 1 | 0.99 | 3 | 1 | 0.9686 | 7 | 1 | 0.9881 |
| | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| | 4 | 7 | 2 | 0.9881 | 23 | 5 | 0.9817 | 7 | 1 | 0.9881 | 27 | 3 | 0.9708 |
| | 6 | 3 | 1 | 0.9912 | 8 | 2 | 0.9718 | 4 | 1 | 0.9566 | 8 | 1 | 0.9718 |
| | 8 | 2 | 1 | 0.982 | 7 | 2 | 0.9881 | 3 | 1 | 0.9686 | 7 | 1 | 0.9881 |

Table 2: Proposed plan for TGLLD with $\theta=2$ and $\lambda=2$ for 50^{th} percentile

| β | $\frac{t_q}{t_0^q}$ | r=5 | | | | | | r=10 | | | | | |
|---------|---------------------|----------------|---|--------|----------------|---|--------|----------------|---|--------|----------------|---|--------|
| | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | |
| | | c | g | Pa | c | g | Pa | c | g | Pa | c | g | Pa |
| 0.25 | 2 | 7 | 3 | 0.9847 | 12 | 3 | 0.9671 | 8 | 2 | 0.9637 | 16 | 2 | 0.9797 |
| | 4 | 1 | 1 | 0.9559 | 3 | 1 | 0.9885 | 2 | 1 | 0.9702 | 5 | 1 | 0.9821 |
| | 6 | 1 | 1 | 0.99 | 2 | 1 | 0.9851 | 1 | 1 | 0.9596 | 3 | 1 | 0.9749 |
| 0.1 | 8 | 1 | 1 | 0.9967 | 1 | 1 | 0.9559 | 1 | 1 | 0.9859 | 2 | 1 | 0.9702 |
| | 2 | 8 | 4 | 0.9637 | 16 | 4 | 0.9797 | 8 | 2 | 0.9637 | 16 | 2 | 0.9797 |
| | 4 | 1 | 1 | 0.9559 | 3 | 1 | 0.9855 | 2 | 1 | 0.9702 | 5 | 1 | 0.9821 |
| 0.05 | 6 | 1 | 1 | 0.99 | 2 | 1 | 0.9851 | 1 | 1 | 0.9596 | 3 | 1 | 0.9749 |
| | 8 | 1 | 1 | 0.9967 | 1 | 1 | 0.9559 | 1 | 1 | 0.9859 | 2 | 1 | 0.9702 |
| | 2 | 8 | 4 | 0.9637 | 19 | 5 | 0.9624 | 8 | 2 | 0.9637 | 23 | 3 | 0.977 |
| 0.01 | 4 | 2 | 2 | 0.9702 | 5 | 2 | 0.9821 | 2 | 1 | 0.9702 | 5 | 1 | 0.9821 |
| | 6 | 1 | 2 | 0.9596 | 2 | 1 | 0.9851 | 1 | 1 | 0.9596 | 3 | 1 | 0.9749 |
| | 8 | 1 | 2 | 0.9859 | 1 | 1 | 0.9559 | 1 | 1 | 0.9859 | 2 | 1 | 0.9702 |
| | 2 | 11 | 6 | 0.9559 | - | - | - | 11 | 3 | 0.9559 | - | - | - |
| | 4 | 3 | 3 | 0.9813 | 5 | 2 | 0.9821 | 4 | 2 | 0.9884 | 5 | 1 | 0.9821 |
| | 6 | 1 | 2 | 0.9596 | 2 | 1 | 0.9851 | 1 | 1 | 0.9596 | 3 | 1 | 0.9749 |
| | 8 | 1 | 2 | 0.9859 | 1 | 1 | 0.9559 | 1 | 1 | 0.9859 | 2 | 1 | 0.9702 |

Table 3: Proposed plan for TGLLD with $\hat{\theta}=1.8138$ and $\hat{\lambda}=2.3381$ for 10^{th} percentile

| β | $\frac{t_q}{t_0^q}$ | r=5 | | | | | | r=10 | | | | | | |
|---------|---------------------|----------------|---|--------|----------------|---|----|----------------|----|----|----------------|---|----|--------|
| | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | |
| | | c | g | Pa | c | g | Pa | c | g | Pa | c | g | Pa | |
| 0.25 | 2 | 8 | 2 | 0.9545 | 0.9545 | - | - | - | 8 | 1 | 0.9545 | - | - | - |
| | 4 | 3 | 1 | 0.9935 | 0.9935 | 8 | 2 | 0.9545 | 4 | 1 | 0.9685 | 8 | 1 | 0.9545 |
| | 6 | 2 | 1 | 0.9946 | 0.9946 | 3 | 1 | 0.9533 | 2 | 1 | 0.9534 | 6 | 1 | 0.9793 |
| 0.1 | 8 | 1 | 1 | 0.9817 | 0.9817 | 3 | 1 | 0.9935 | 2 | 1 | 0.9915 | 4 | 1 | 0.9685 |
| | 2 | 12 | 3 | 0.9736 | 0.9736 | - | - | - | 15 | 2 | 0.9504 | - | - | - |
| | 4 | 3 | 1 | 0.9935 | 0.9935 | 8 | 2 | 0.9545 | 4 | 1 | 0.9685 | 8 | 1 | 0.9545 |
| 0.05 | 6 | 2 | 1 | 0.9946 | 0.9946 | 3 | 1 | 0.9533 | 2 | 1 | 0.9534 | 6 | 1 | 0.9793 |
| | 8 | 1 | 1 | 0.9817 | 0.9817 | 3 | 1 | 0.9935 | 2 | 1 | 0.9915 | 4 | 1 | 0.9685 |
| | 2 | 15 | 4 | 0.9504 | 0.9504 | - | - | - | 15 | 2 | 0.9504 | - | - | - |
| 0.01 | 4 | 3 | 1 | 0.9935 | 0.9935 | 8 | 2 | 0.9545 | 4 | 1 | 0.9685 | 8 | 1 | 0.9545 |
| | 6 | 2 | 1 | 0.9946 | 0.9946 | 3 | 1 | 0.9533 | 2 | 1 | 0.9534 | 6 | 1 | 0.9793 |
| | 8 | 1 | 1 | 0.9817 | 0.9817 | 3 | 1 | 0.9935 | 2 | 1 | 0.9915 | 4 | 1 | 0.9685 |
| 0.01 | 2 | 19 | 5 | 0.9716 | 0.9716 | - | - | - | 22 | 3 | 0.9579 | - | - | - |
| | 4 | 4 | 2 | 0.9928 | 0.9928 | 8 | 2 | 0.9545 | 4 | 1 | 0.9685 | 8 | 1 | 0.9545 |
| | 6 | 3 | 2 | 0.9928 | 0.9928 | 3 | 1 | 0.9533 | 2 | 1 | 0.9534 | 6 | 1 | 0.9793 |
| | 8 | 1 | 1 | 0.9817 | 0.9817 | 3 | 1 | 0.9935 | 2 | 1 | 0.9915 | 4 | 1 | 0.9685 |

Table 4: Proposed plan for TGLLD with $\hat{\theta}=1.8138$ and $\hat{\lambda}=2.3381$ for 50^{th} percentile

| β | $\frac{t_q}{t_0^q}$ | r=5 | | | | | | r=10 | | | | | | |
|---------|---------------------|----------------|---|--------|----------------|----|----|----------------|---|----|----------------|----|----|--------|
| | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | $\delta_q=0.5$ | | | $\delta_q=1.0$ | | | |
| | | c | g | Pa | c | g | Pa | c | g | Pa | c | g | Pa | |
| 0.25 | 2 | 3 | 2 | 0.9635 | 0.9635 | 7 | 2 | 0.9614 | 3 | 1 | 0.9635 | 7 | 1 | 0.9614 |
| | 4 | 1 | 1 | 0.9918 | 0.9918 | 2 | 1 | 0.9796 | 1 | 1 | 0.9666 | 3 | 1 | 0.9635 |
| | 6 | 1 | 1 | 0.9987 | 0.9987 | 1 | 1 | 0.9716 | 1 | 1 | 0.9943 | 2 | 1 | 0.984 |
| 0.1 | 8 | 0 | 1 | 0.9707 | 0.9707 | 1 | 1 | 0.9918 | 1 | 1 | 0.9985 | 1 | 1 | 0.9666 |
| | 2 | 4 | 3 | 0.9569 | 0.9569 | 10 | 3 | 0.9611 | 5 | 2 | 0.9549 | 13 | 2 | 0.9648 |
| | 4 | 1 | 2 | 0.9666 | 0.9666 | 2 | 1 | 0.9796 | 1 | 1 | 0.9666 | 3 | 1 | 0.9635 |
| 0.05 | 6 | 1 | 2 | 0.9943 | 0.9943 | 1 | 1 | 0.9716 | 1 | 1 | 0.9943 | 2 | 1 | 0.984 |
| | 8 | 0 | 1 | 0.9707 | 0.9707 | 1 | 1 | 0.9918 | 1 | 1 | 0.9985 | 1 | 1 | 0.9666 |
| | 2 | 5 | 4 | 0.9549 | 0.9549 | 10 | 3 | 0.9611 | 5 | 2 | 0.9549 | 13 | 2 | 0.9648 |
| 0.01 | 4 | 1 | 2 | 0.9666 | 0.9666 | 2 | 1 | 0.9796 | 1 | 1 | 0.9666 | 3 | 1 | 0.9635 |
| | 6 | 1 | 2 | 0.9943 | 0.9943 | 1 | 1 | 0.9716 | 1 | 1 | 0.9943 | 2 | 1 | 0.984 |
| | 8 | 0 | 1 | 0.9707 | 0.9707 | 1 | 1 | 0.9918 | 1 | 1 | 0.9985 | 1 | 1 | 0.9666 |
| 0.01 | 2 | 7 | 6 | 0.9569 | 0.9569 | 13 | 4 | 0.9648 | 7 | 3 | 0.9564 | 13 | 2 | 0.9648 |
| | 4 | 2 | 3 | 0.9911 | 0.9911 | 3 | 2 | 0.9635 | 2 | 2 | 0.98 | 3 | 1 | 0.9635 |
| | 6 | 1 | 3 | 0.9873 | 0.9873 | 1 | 1 | 0.9716 | 2 | 1 | 0.9778 | 2 | 1 | 0.984 |
| | 8 | 1 | 3 | 0.9965 | 0.9965 | 1 | 1 | 0.9918 | 2 | 1 | 0.9938 | 1 | 1 | 0.9666 |