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Dynamic Multi State System Reliability Analysis of Power Generating Systems using Lz-transform

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Abstract. Multi-state systems have been widely applied in engineering in recent times. Power generating and supply systems, communication systems, transportation systems etc are modeled by using Multi-State Systems (MSS). In this article we propose a technique based on Lz transform method, avoiding the curse of dimensionality of stochastic process approach which is often used for the reliability analysis of MSS. This method can drastically minimize the computational burden for dynamic reliability assessment of repairable multi state system assuming variable failure rates and repair rates of components of the system. We illustrate this method for the availability evaluation of a power station based on a real data set.

1. Introduction

In the usual binary reliability models we assume the system and its components to be either in a perfectly functioning state or in a completely down state. However this assumption may not be adequate in several real life situations. There are intermediate states between perfectly functioning state and completely failed state. Hence Multi State System (MSS) reliability models where the system may rather have more than two states of performance between perfect working and complete failure, have been more realistically used. Such systems can conduct their task with partial performance. Failures of some system elements lead only to the lowering of system performance. The basic concept and further developments of binary system reliability theory were dealt in [1], [2]. The basic concepts of MSS, tools for MSS reliability assessment and optimization and application problems were discussed in [8]. Multi state with degrading components and the reliability evaluation of large systems were emphasized in [6]. A comprehensive exposition of system reliability theory has been presented in [12].

Generally stochastic process method is used for evaluating the MSS reliability measures. Disadvantage of this method is that the stochastic process models are very difficult for application to real world MSS consisting of many elements with many states. During recent years a specific approach named "Universal Generating Function (UGF)" procedure has been used in MSS reliability analysis [7], [8]. This approach was introduced by Ushakov [14], [15]. UGF technique is essentially based on moment generating functions and it is a mathematical concept for random variables. UGF plays a crucial role in the steady state analysis of MSS. For a Discrete State Continuous Time (DSCT) Markov process a special transform called "Lz transform" which is an extension of UGF technique is introduced [10]. Multi state models are extensively

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used in the field of power system reliability analysis [3]. Lz transform method is suggested [10, p.79-85] for dynamic reliability analysis of some MSS. In [4] Lz transform method is applied to dynamic reliability analysis of some MSS. Lz transform method has been demonstrated in [11] for short term evaluation of a power generating system and several important indices of the system have been evaluated. An instantaneous availability model for multi state repairable system with common bus performance sharing has been proposed in [5]. In this paper the idea used in [5] and [11] have been adapted and employed in the case of a power generating system of n components of multiple states connected in parallel.

Evidently total output of the system in the above power generating system model is equal to the sum of the output of the individual components. We determine Lz transform for each individual element after solving differential equations for Markov model of each system element in order to obtain state probabilities as functions of time. By using Ushakov's UGF we get Lz transform for the entire system output and also determine the reliability measures of corresponding power system. Application of this approach on power system reliability analysis is illustrated by a numerical example. On these lines six sections in this paper are developed and presented.

2. Model Description

Assumptions

1. The power generating system may have many levels of degradation which vary from perfect functioning to complete failure.
2. The system might fail from any 'up' state to its 'down' states and it is minimally repaired.
3. The components of the system might fail independently and they are operated on continuous basis.
4. The components of the system can be repaired independently.
5. The Failure rates and repair rates from one state to other state are varying for each component of the system.

Consider a system with n components each having $1, 2, ...k_j$ states where k_j is the best functioning state and 1 is the worst state. The state space of the component of the system is
S $=\{1, 2, ...k_j\}$.

3. Methodology

Consider a power station with n generators , having states $k_1, k_2, ..., k_n$ respectively. Markov model of whole power station will have $k_1 \times k_2 \times ... \times k_n$ states. This model can be analysed for finding reliability indices of the power station. If conventional stochastic process approach [8] is applied, it will require huge effort even for relatively small n and $k_j, j = 1, 2, ..., n$. UGF technique can be applied for avoiding this dimension- damnation problem. Now UGF being defined for random variables, one needs to consider only the steady state behaviour of the power system. Here Lz transform method is applied for finding reliability indices for short term reliability evaluation of a power station consisting of numerous different generating units.

Consider a discrete state continuous time (DSCT) Markov process $X(t) \in \{x_1, x_2 \dots x_k\}$ which has k possible states $i, (i = 1, 2 \dots k)$ where performance level associated with state i is x_i.

This Markov process is completely defined by set of possible states $X = \{x_1, x_2 \dots x_k\}$, transition intensities matrix $A = (a_{ij}(t)), \quad i, j = 1, 2 \dots k$ and probability distribution of initial states. Probability distribution of initial states is represented by corresponding set

$$p_0 = [p_{10} = Pr\{X(0) = x_1\}, p_{20} = Pr\{X(0) = x_2\},$$
$$\dots\dots\dots p_{k0} = Pr\{X(0) = x_k\}].$$

In general case j^{th} component of a power generating system $(j \in \{1, 2 \dots n\})$ have k_j different states corresponding to different performances. It is represented by the set $g_j = \{g_{j1}, g_{j2}, \dots g_{jk_j}\}$ where g_{ji} is the performance level of component j in the state i, $(i \in \{1, 2 \dots k_j\}$ and $j \in \{1, 2 \dots n\})$.

According to [10] Lz transform of a DSCT Markov process X(t) is a function defined as follows

$$Lz\{X(t)\} = \sum_{i=1}^{k} p_i(t) z^{g_i} \tag{1}$$

where $p_i(t)$ is a probability that the process is in state i at time instant $t \geq 0$ for a given initial states probability distribution p_0 and z in general case is a complex variable.

At first stage a Markov model should be constructed for each multi state element in MSS. Solving the following system of linear differential equation of j^{th} component [13].

$$\begin{aligned}
\frac{d}{dt} p_{j1}(t) &= a_{11}(t)p_{j1}(t) + a_{12}(t)p_{j2}(t) + \dots \\
&\quad + a_{1k_j}(t)p_{jk_j}(t) \\
\frac{d}{dt} p_{j2}(t) &= a_{21}(t)p_{j1}(t) + a_{22}(t)p_{j2}(t) + \dots \\
&\quad + a_{2k_j}(t)p_{jk_j}(t) \\
&\dots\dots\dots\dots\dots\dots\dots\dots \\
\frac{d}{dt} p_{jk_j}(t) &= a_{k_j1}(t)p_{j1}(t) + a_{k_j2}(t)p_{j2}(t) + \dots \\
&\quad + a_{k_jk_j}(t)p_{jk_j}(t)
\end{aligned}$$

under the given initial conditions $p_0 = \{p_{10}, p_{20} \dots p_{k_j0}\}$ we get the probabilities $p_{ji}(t), i = 1.2...k_j, j = 1, 2, ...n$ The individual Lz transform for each component j can be obtained by the formula

$$Lz\{G_j(t)\} = \sum_{i=1}^{k_j} P_{ji}(t) z^{g_{ji}}, j = 1, 2...n \tag{2}$$

Lz transform of whole MSS can be obtained based on Lz transform for each component and system structure function f. By applying Ushakov's operator Ω_f over all Lz transform of individual elements we get the resulting Lz transform, $Lz\{G(t)\}$ [10] linked with output performance stochastic process $G(t)$ of the whole MSS. Employing Ushakov's Universal Generating Operator (UGO) to all individual Lz transforms $Lz\{G_j(t)\}$ over all time point $t \geq 0$ we can obtain

$$Lz\{G(t)\} = \Omega_f\{Lz\{G_1(t)\}, Lz\{G_2(t)\}, \dots Lz\{G_n(t)\}\} \tag{3}$$

Ushakov's operator is well defined for many different structure functions f [9]. By using technique of Lz transform we can drastically minimize computational burden and $Lz\{G(t)\}$ is associated with the output performance of the entire MSS. Multi state system reliability measure can be obtained from the resulting Lz transform $Lz\{G(t)\}$, as summarized in the next section.

4. System Reliability Measures

If Lz-transform

$$Lz\{G(t)\} = \sum_{k=1}^{K} p_k(t) z^{g_k} \tag{4}$$

of the entire MSS's output stochastic process
$G(t) \in \{g_1, g_2, \dots g_k\}$ is known, then important system reliability measures (Ref. [5]) can be easily obtained.

The power station availability for demand level w is defined as system ability to provide power supply to consumers with summarized load w. That is, the power station should be in states with generating capacity more or equal w.

Therefore the system availability for the constant demand w at instant $t \geq 0$ is given by

$$A_w(t) = \sum_{g_k \geq w} p_k(t). \tag{5}$$

In order to find MSS instantaneous availability we should summarize all probabilities in Lz transform from terms where powers of z are greater or equal to demand w.

Loss of Load probability ($LOLP_w$) for a given level w is then obtained as

$$LOLP_w(t) = 1 - A_w(t). \tag{6}$$

The expected generating capacity deficiency (ECD_w) of the system is given by

$$ECD_w(t) = \sum_{k=1}^{K} p_k(t)(w - g_k)I_{(w-g_k)} \tag{7}$$

where

$$I_{w-g_K} = \begin{cases} 1 & \text{if } w - g_k > 0 \\ 0 & \text{if } w - g_k \leq 0 \end{cases}$$

.

Reliability measures for a power system depend strongly on initial states of units.

5. Availability Evaluation of a power station with six generating units connected in parallel

In this section we apply the methods presented in the foregoing section to carry out the availability analysis based on the data collected from Kuttiady Hydro Electric Project, governed by Kerala State Electricity Board(KSEB) under Govt. of Kerala, located at Kakkayam, Kozhikode district. With an installed capacity 75 MW(3 generators each with 25 MW) the Kuttiady power station was commissioned on 30-09-1972. The next generator with installed capacity 50MW was commissioned on 27-01-2001. Last two generators with installed capacity each 50 MW were commissioned on 30-10-2010 and 10-11-2010. Six generating units are connected parallel ie, total output of the system is equal to the sum of the output of the the generating units. Total capacity of the power station is 225MW. The generation of the power station is controlled by state Load Despatch Centre, a functional unit of KSEB, which is the apex body to ensure integrated operation of the power system in Kerala. According to the centre the production of generators are categorized into three - either in full generation mode or half generation mode or zero generation mode. In our Markov model transitions of states occur due to failures and repairs. The states and outputs of Generator 1,2, and 3 (G1,G2 and G3) are 1(0 MW), 2(12.5 MW)and 3(25 MW). States and outputs of Generator 4,5 and 6 (G4, G5 and G6) are 1(0 MW), 2(25 MW)and 3(50 MW).

Following the terminology used earlier, in this case we have a Markov model and we shall apply the Lz transform technique to evaluate the availability of the power system.

Transition intensities a_{ij} drawn up as a matrix, called infinitesimal generator of the process is given by

$$A = |a_{ij}| = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k_j} \\ a_{21} & a_{22} & \dots & a_{2k_j} \\ \vdots & \vdots & \vdots & \vdots \\ a_{k_j1} & a_{k_j2} & \dots & a_{k_jk_j} \end{pmatrix}$$

$$= \begin{pmatrix} -\mu_{12}^j & \mu_{12}^j & 0 \\ \lambda_{21}^j & -(\lambda_{21}^j + \mu_{23}^j) & \mu_{23}^j \\ \lambda_{31}^j & \lambda_{32}^j & -(\lambda_{31}^j + \lambda_{32}^j) \end{pmatrix}, \ j = 1, 2, 3, 4, 5, 6$$

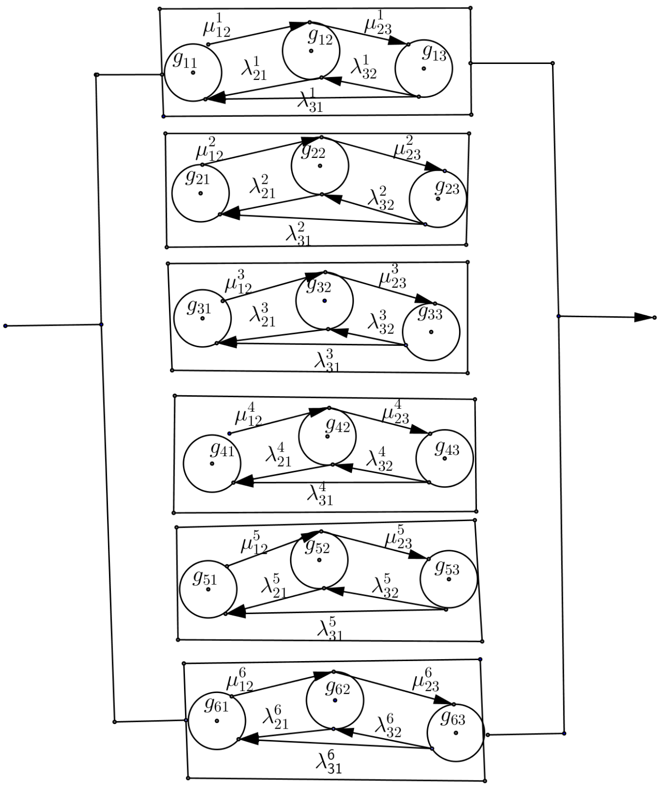

Figure 1: State space diagram of the power generating system with six generators

Table 1: Transition rates of the generators per hour(h^{-1}) are calculated from the collected data and are given the table below

Generator	μ_{12}	μ_{23}	λ_{21}	λ_{32}	λ_{31}
G1	7.1×10^{-2}	6.4×10^{-2}	3×10^{-3}	6.7×10^{-2}	3.3×10^{-3}
G2	7.3×10^{-2}	6.5×10^{-2}	3×10^{-3}	6.8×10^{-2}	3.3×10^{-3}
G3	7.4×10^{-2}	6.1×10^{-2}	3×10^{-3}	6.2×10^{-2}	3.3×10^{-3}
G4	7.8×10^{-2}	6.6×10^{-2}	3.3×10^{-3}	6.9×10^{-2}	3×10^{-3}
G5	7.8×10^{-2}	6.4×10^{-2}	3.4×10^{-3}	6.7×10^{-2}	3×10^{-3}
G6	7.9×10^{-2}	6.4×10^{-2}	3.3×10^{-3}	6.7×10^{-2}	3×10^{-3}

We have Kolmogorov forward equation in matrix term

$$\frac{d}{dt} p_j(t) = p_j(t)\ A. \tag{8}$$

For elements $j = 1, 2, 3$;

$$g_j = \{g_{j1}, g_{j2}, g_{j3}\} = \{0, 12.5, 25\}$$
$$p_j(t) = \{p_{j1}(t), p_{j2}(t), p_{j3}(t)\}$$

$$\begin{aligned}
\frac{d}{dt}p_{j1}(t) &= -\mu^j_{12}(t)p_{j1}(t) + \lambda^j_{21}p_{j2}(t) + \lambda^j_{31}p_{j3}(t) \\
\frac{d}{dt}p_{j2}(t) &= \mu^j_{12}(t)p_{j1}(t) - (\lambda^j_{21} + \mu^j_{23})p_{j2}(t) + \lambda^j_{32}p_{j3}(t) \\
\frac{d}{dt}p_{j3}(t) &= -\mu^j_{23}(t)p_{j2}(t) - (\lambda^j_{31} + \lambda^j_{32})p_{j3})(t)
\end{aligned}$$

Initial conditions are $p_{j3}(t) = 1, p_{j2}(t) = p_{j1}(t) = 0$.

For elements $j = 4, 5, 6$;

$$g_j = \{g_{j1}, g_{j2}, g_{j3}\} = \{0, 25, 50\}$$
$$p_j(t) == \{p_{j1}(t), p_{j2}(t), p_{j3}(t)\}$$

$$\begin{aligned}
\frac{d}{dt}p_{j1}(t) &= -\mu^j_{12}(t)p_{j1}(t) + \lambda^j_{21}p_{j2}(t) + \lambda^j_{31}p_{j3}(t) \\
\frac{d}{dt}p_{j2}(t) &= \mu^j_{12}(t)p_{j1}(t) - (\lambda^j_{21} + \mu^j_{23})p_{j2}(t) + \lambda^j_{32}p_{j3}(t) \\
\frac{d}{dt}p_{j3}(t) &= -\mu^j_{23}(t)p_{j2}(t) - (\lambda^j_{31} + \lambda^j_{32})p_{j3}(t)
\end{aligned}$$

Initial conditions are $p_{j3}(t) = 1, p_{j2}(t) = p_{j1}(t) = 0$.

After solving six separate system of differential equations under the given initial conditions using MATLAB we get the state probabilities $p_j(t)$ for $j = 1, 2, 3, 4, 5, 6$. Having the sets $g_j, p_j(t)$ we can define Lz transforms for each individual element j as follows.

$$\begin{aligned}
Lz\{G_1(t)\} &= p_{11}(t)z^{g_{11}} + p_{12}(t)z^{g_{12}} + p_{13}(t)z^{g_{13}} \\
&= p_{11}(t)z^0 + p_{12}(t)z^{12.5} + p_{13}(t)z^{25} \\
Lz\{G_2(t)\} &= p_{21}(t)z^{g_{21}} + p_{22}(t)z^{g_{22}} + p_{23}(t)z^{g_{23}} \\
&= p_{21}(t)z^0 + p_{22}(t)z^{12.5} + p_{23}(t)z^{25} \\
Lz\{G_3(t)\} &= p_{31}(t)z^{g_{31}} + p_{32}(t)z^{g_{32}} + P_{33}(t)z^{g_{33}} \\
&= p_{31}(t)z^0 + p_{32}(t)z^{12.5} + p_{33}(t)z^{25} \\
Lz\{G_4(t)\} &= p_{41}(t)z^{g_{41}} + p_{42}(t)z^{g_{42}} + p_{43}(t)z^{g_{43}} \\
&= p_{41}(t)z^0 + p_{42}(t)z^{25} + p_{43}(t)z^{50} \\
Lz\{G_5(t)\} &= p_{51}(t)z^{g_{51}} + p_{52}(t)z^{g_{52}} + p_{53}(t)z^{g_{53}} \\
&= p_{51}(t)z^0 + p_{52}(t)z^{25} + p_{53}(t)z^{50} \\
Lz\{G_6(t)\} &= p_{61}(t)z^{g_{61}} + p_{62}(t)z^{g_{62}} + p_{63}(t)z^{g_{63}} \\
&= p_{61}(t)z^0 + p_{62}(t)z^{25} + p_{63}(t)z^{50}
\end{aligned}$$

Using composition operator Ω_f par [9, p.167] for MSS elements 1,2,3,4,5 and 6 connected in parallel we get the Lz transform

$$\begin{aligned}
Lz\{G(t)\} = \Omega_f par\{&p_{11}(t)z^0 + p_{12}(t)z^{12.5} + p_{13}(t)z^{25}, \\
&p_{21}(t)z^0 + p_{22}(t)z^{12.5} + p_{23}(t)z^{25}, \\
&p_{31}(t)z^0 + p_{32}(t)z^{12.5} + p_{33}(t)z^{25}, \\
&p_{41}(t)z^0 + p_{42}(t)z^{25} + p_{43}(t)z^{50}, \\
&p_{61}(t)z^0 + p_{62}(t)z^{25} + p_{63}(t)z^{50}\}
\end{aligned}$$

Thus

$$Lz\{G(t)\} = \sum_{k=1}^{19} p_k(t) z^{g_k} \tag{9}$$

The state probabilities of the components of this power generating system can be calculated by solving system of differential equations of each component under given initial conditions with the help of MATLAB. For Element 1:

$$p_{11}(t) = 0.0405 - 0.4445 \exp\{-0.0743t\} + 0.404 \exp\{-0.134t\}$$
$$p_{12}(t) = 0.9594 - 0.057 \exp\{-0.0743t\} - 0.9024 \exp\{-0.134t\}$$
$$p_{13}(t) = 0.0000005 + 0.5015753 \exp\{-0.0743t\} + 0.4984242 \exp\{-0.134t\}$$

For Element 2:

$$p_{21}(t) = 0.04 - 0.43 \exp\{-0.0763t\} + 0.39 \exp\{-0.136t\}$$
$$p_{22}(t) = 0.96 - 0.069 \exp\{-0.0763t\} - 0.891 \exp\{-0.136t\}$$
$$p_{23}(t) = 0.000002 + 0.501940 \exp\{-0.0763t\} + 0.498058 \exp\{-0.136t\}$$

For Element 3:

$$p_{31}(t) = 0.0418 - 0.3495 \exp\{-0.0771t\} + 0.3077 \exp\{-0.1261t\}$$
$$p_{32}(t) = 0.956 - 0.163 \exp\{-0.0771t\} + 0.793 \exp\{-0.1261t\}$$
$$p_{33}(t) = 0.0018 + 0.5131 \exp\{-0.0771t\} + 0.4851 \exp\{-0.1261t\}$$

For Element 4:

$$p_{41}(t) = 0.04 - 0.39 \exp\{-0.081t\} + 0.35 \exp\{-0.1383t\}$$
$$p_{42}(t) = 0.957 - 0.116 \exp\{-0.081t\} - 0.841 \exp\{-0.1383t\}$$
$$p_{43}(t) = 0.003 + 0.504 \exp\{-0.081t\} + 0.493 \exp\{-0.1383t\}$$

For Element 5:

$$p_{51}(t) = 0.039 - 0.364 \exp\{-0.0809t\} + 0.325 \exp\{-0.1345t\}$$
$$p_{52}(t) = 0.955 - 0.138 \exp\{-0.0809t\} - 0.817 \exp\{-0.1345t\}$$
$$p_{53}(t) = 0.0052 + 0.5034 \exp\{-0.0809t\} + 0.4914 \exp\{-0.1345t\}$$

For Element 6:

$$P_{61}(t) = 0.038 - 0.352 \exp\{-0.0819t\} + 0.314 \exp\{-0.1344t\}$$
$$P_{62}(t) = 0.957 - 0.154 \exp\{-0.0819t\} - 0.803 \exp\{-0.1344t\}$$
$$P_{63}(t) = 0.0041 + 0.5058 \exp\{-0.0819t\} + 0.4901 \exp\{-0.1344t\}$$

Based on the resulting Lz transform Lz{G(t)} of the entire MSS ,we can obtain instantaneous availability for the given demand w.

According to Load Despatch Centre average demand for a particular month of this power station is $w = 108.4$ MW. The power station availability for this demand level is given by

$$A_{108.4}(t) = \sum_{g_k \geq 108.4} p_k(t). \tag{10}$$

The MSS instantaneous availability A(t) of the power system is calculated for different hours and the computed values are presented in the figure 2.

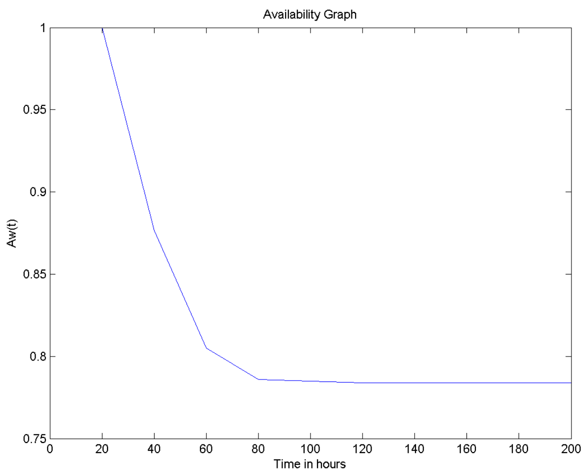

Figure 2: Graph of power system Availability (for the demand w=108.4 MW) as a function of time

The figure shows that instantaneous availability of the power system is one up to few hours (0 to 20 hours) and decreases after few hours and later eventually attains a stable value.

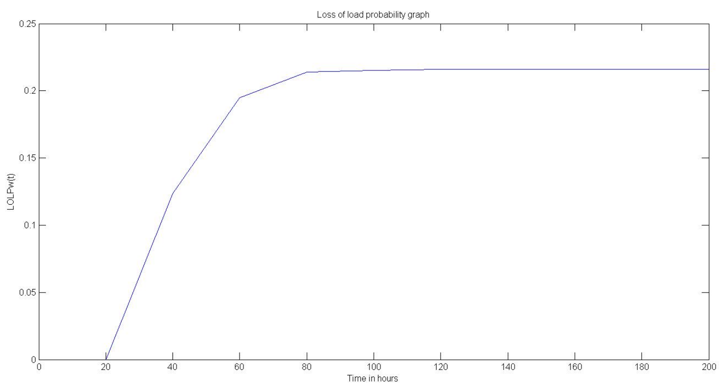

Figure 3: Graph of loss of load probability (for the demand w=108.4 MW) as a function of time

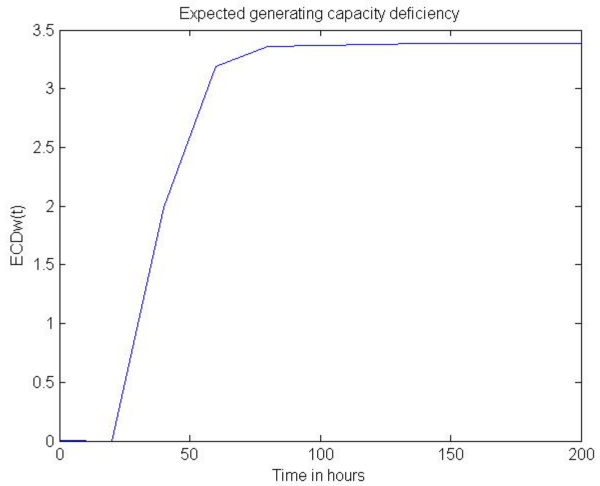

Figure 4: Graph of the expected generating capacity deficiency (for the demand w=108.4 MW) as a function of time

6. Conclusion

In this paper Lz transform for discrete state continuous time Markov process is presented for a power generating system of multiple components with multiple states connected in parallel. The methods are employed as a case study for a power station with six generating units connected in parallel. Lz transform is obtained using simple algebra and it is proved to be a very effective method. The idea of this paper supports the engineering decision making by providing required availability measure for such complex multi-state system with multiple components.

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