

THREE CLASS ATTRIBUTES DOUBLE SAMPLING PLAN INDEXED THROUGH MAXIMUM ALLOWABLE AVERAGE OUTGOING QUALITY

R. Radhakrishnan

Reader in Statistics, PSG College of Arts & Science
Coimbatore 641 014, India.

Email: *rkrishnan_cbe@yahoo.com*

S. Ravi Sankar

Lecturer in Statistics, CMS College of Science & Commerce
Coimbatore 641 006.

Email: *ks.ravisankar@yahoo.com*

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Abstract

This paper presents 3-class attributes double sampling plan indexed through Maximum Allowable Average Outgoing Quality (MAAOQ). This 3-class sampling procedure classifies items in a lot under consideration into three categories of quality good, marginal and bad. Two proportions of marginal and bad items define the lot quality. A table is constructed for the easy selection of 3-class attributes double sampling plan for the specified MAAOQ. The procedure for construction of the table is also given in detail. The selection of the plan for the specified MAAOQ is explained with example.

Key words and Phrases. Trinomial Distribution, Operating Characteristic, Double Sampling Plan, Three Class Attributes Plan, Maximum Allowable Average Outgoing Quality, Marginal Quality.

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1 Introduction

The Maximum Allowable Percent Defective (MAPD) is the proportion defective at the inflection point of the OC curve. Soundararajan [10] proposed a procedure for the selection of a single sampling plan indexed through MAPD. Average Outgoing Quality (AOQ) is the quality level that the customer receives in the long run. MAAOQ is the AOQ value corresponding to the quality level MAPD. Suresh and Ramkumar [11] have proposed a procedure for the selection of the parameter for a 2-class single sampling plan using the MAPD as the standard quality and the MAAOQ as an average outgoing quality. This procedure protects the interest of the consumer in terms of incoming and out-

going quality.

Acceptance sampling has been generally carried out using either two class attribute plans or variable plans. These plans classify a lot of items as acceptable or non-acceptable, the proportion non-acceptable being called the proportion defective. These plans attempt solely to control the proportion defective accepted. But these plans provide no information on the proportion of items in the area around the quality limit, that is, they do not distinguish between a near miss item and an extremely bad one. Bray et al [1] introduced a special type of plan known as 3-class attributes plan. It considers a near miss item as marginal item. This procedure classifies three categories of quality good, marginal and bad. The two proportions of marginal and bad items define the lot quality.

Clements [2] showed that the 3-class attributes plans are more efficient than the conventional 2-class attributes plans. Suresh and Ravi Sankar [12] developed procedure for Double Sampling Plan for three attribute classes and Suresh et al [13] developed a procedure for Link Sampling Plan for three attribute classes. Radhakrishnan and Ravi Sankar [4, 6, 7, 8] developed procedures for the selection of 3-class attributes single sampling plans indexed through the parameters such as AOQL, AQL, IQL and MAPD. Radhakrishnan and Ravi Sankar [9] developed a procedure for the selection of 3-class attributes double sampling plans indexed through fixed points on OC surface. Radhakrishnan and Ravi Sankar [5] constructed 3-class attributes link-sampling plans indexed through AQL. Radhakrishnan [3] constructed three-class attributes single sampling plans through Six Sigma Quality Levels. In this paper, an attempt is made to construct 3-class attributes double sampling plan indexed through MAAOQ.

2 Glossary of Symbols

n_1 First Sample size

n_2 Second Sample size

p_g Proportion of good quality units.

p_M Proportion of marginal quality units.

p_b Proportion of bad quality units.

d_{11} Total number of marginal and bad units found in the first sample of size n_1 .

d_{12} Number of bad units found in the first sample of size n_1 .

- d_{21} Total number of marginal and bad units found in the second sample of size n_2 .
- d_{12} Number of bad units found in the second sample of size n_2 .
- c_1 Acceptance number for the marginal and bad units in the first sample.
- c_2 Acceptance number for the bad units in the first sample.
- $c_1 + b_1$ Acceptance number for the marginal and bad units in the combined sample.
- $c_2 + b_2$ Acceptance number for the bad units in the combined sample.
- p^* Maximum Allowable Percent Defective (MAPD)
- $\text{Pa}(p_M, p_b)$ Probability of acceptance for the given quality level (p_M, p_b)
- $\text{DSP3}(n_1, n_2, c_1, b_1, c_2, b_2)$ 3-class attributes double sampling plan with parameters n_1, n_2, c_1, b_1, c_2 and b_2 .

3 Operating procedure for the DSP3($n_1, n_2, c_1, b_1, c_2, b_2$)

According to Suresh and RaviSankar [11], the operating procedure for three class attributes Double sampling plan DSP3 ($n_1, n_2, c_1, b_1, c_2, b_2$) is given below:

- Step 1** Inspect a random sample of size n_1 .
- Step 2** Count the total number of marginal and bad items (d_{11}) and the number of bad items (d_{12}) found in the first sample.
- Step 3** If $d_{11} \leq c_1$ and $d_{12} \leq c_2$, accept the lot; If $d_{11} > c_1 + b_1$ or $d_{12} > c_2 + b_2$ reject the lot; If either (a) $c_1 < d_{11} \leq c_1 + b_1$ and $d_{12} \leq c_2$ (or) (b) $d_{11} \leq c_1 + b_1$ and $c_2 < d_{12} \leq c_2 + b_2$ go to step 4.
- Step 4** Inspect another sample of size n_2 from the same lot. Count the total number of marginal and bad items (d_{21}) and the number of bad items (d_{22}) in the second sample.
- Step 5** Let $D_1 = d_{11} + d_{21}$, the total number of marginal and bad items in the combined sample.

Let $D_2 = d_{12} + d_{22}$, the total number of bad items in the combined sample.
If $D_1 \leq c_1 + b_1$ and $D_2 \leq c_2 + b_2$, then accept the lot; otherwise reject the lot.

4 Operating Characteristic Function

The operating characteristic (OC) function of the plan DSP3 ($n_1, n_2, c_1, b_1, c_2, b_2$) is

$$\begin{aligned} \text{Pa}(p_M, p_b) &= \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} m_{ij}(n_1) \\ &+ \left[\sum_{j=0}^{c_2} \sum_{i=c_1+1-j}^{c_1+b_1-j} m_{ij}(n_1) + \sum_{j=c_2+1}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} m_{ij}(n_1) \right] \sum_{j=0}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} m_{ij}(n_2) \\ &\text{where } m_{ij}(n_r) = \frac{n_r}{(n_r - i - j)! i! j!} p_g^{n_r-i-j} p_M^i p_b^j; \quad (r = 1, 2) \quad (4.1) \end{aligned}$$

It is based on the trinomial probability distribution, which is a particular case of a multinomial distribution. Note that the graph for this OC function of DSP3($n_1, n_2, c_1, b_1, c_2, b_2$) is an OC surface with the probability of acceptance $\text{Pa}(p_M, p_b)$ plotted against the two quality parameters p_M and p_b .

5 Maximum Allowable Percent Defective (MAPD)

In case of 2-class attribute plans, the Maximum Allowable Percent Defective (MAPD) is the quality level corresponding to the inflection point of the OC curve. The inflection point is a point at which $\text{Pa}''(p_b) = 0$ and $\text{Pa}'''(p_b) \neq 0$. In a 3-class attribute plan, there are many such inflection points on the OC surface and for each proportion of marginal units (p_M) there is a p_b value at an inflection point and the sum of these two proportions is considered as the MAPD ($p^* = p_M + p_b$).

6 Maximum Allowable Average Outgoing Quality (MAAOQ)

The AOQ function of the single sampling plan is given by

$$\text{AOQ} = p \cdot \text{Pa}(p) \quad (6.1)$$

where $p = p_M + p_b = (1 + k) \cdot p_b$, ($0 < k < 1$)

The MAAOQ of sampling plan is defined as average outgoing quality (AOQ) at the MAPD.

$$\begin{aligned} \text{i.e., MAAOQ} &= \text{AOQ at } p = p^* \\ &= p^* \cdot \text{Pa}(p^*) \\ &= (p_M + p_b) \cdot \text{Pa}(p_M, p_b). \end{aligned} \quad (6.2)$$

7 Designing DSP3 ($n_1, n_2, c_1, b_1, c_2, b_2$) Indexed through MAAOQ

By taking $p_M = k p_b$, ($0 < k < 1$), the OC function defined in equation (4.1) is converted into a function of p_b which is given by

$$\text{Pa}(p_b) = \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} u_{ij}(n_1) + \left[\sum_{j=0}^{c_2} \sum_{i=c_1+1-j}^{c_1+b_1-j} u_{ij}(n_1) + \sum_{j=c_2+1}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_1) \right] \\ + \sum_{j=0}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_2) \quad (7.1)$$

$$\text{where } u_{ij}(n_r) = \frac{n_r!}{(n_r - i - j)! i! j!} k^i p_g^{n_r - i - j} p_b^{i+j}; \quad r = 1, 2$$

$$p_g = 1 - (1 + k)p_b$$

$$= T_{11} + [T_{21} + T_{22}] \cdot T_{23}$$

Differentiating (7.1) with respect to p_b ,

$$\text{Pa}'(p_b) = T'_{11} + [T_{21} + T_{22}]T'_{23} + [T'_{21} + T'_{22}]T_{23} \quad (7.2)$$

$$\text{where } T'_{11} = \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} u_{ij}(n_1) \left[\frac{i+j}{p_b} - \frac{n_1 - (i+j)(1+k)}{p_g} \right] \\ T'_{21} = \sum_{j=0}^{c_2} \sum_{i=c_1+1-j}^{c_1+b_1-j} u_{ij}(n_1) \left[\frac{i+j}{p_b} - \frac{n_1 - (i+j)(1+k)}{p_g} \right] \\ T'_{22} = \sum_{j=c_2+1}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_1) \left[\frac{i+j}{p_b} - \frac{n_1 - (i+j)(1+k)}{p_g} \right] \\ T'_{23} = \sum_{j=0}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_2) \left[\frac{i+j}{p_b} - \frac{n_2 - (i+j)(1+k)}{p_g} \right]$$

where

$$u_{ij}(n_r) = \frac{n_r!}{(n_r - i - j)! i! j!} k^i p_g^{n_r - i - j} p_b^{i+j}; \quad r = 1, 2$$

Again differentiating (7.2) with respect to p_b ,

$$\text{Pa}''(p_b) = T''_{11} + [T_{21} + T_{22}]T''_{23} + 2[T'_{21} + T'_{22}]T'_{23} + [T''_{21} + T''_{22}]T_{23} \quad (7.3)$$

where

$$\begin{aligned}
T''_{11} &= \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} u_{ij}(n_1) \left[\frac{(i+j)(i+j-1)}{P_b^2} - \frac{2(1+k)(i+j)(n_1-i-j)}{P_g P_b} \right. \\
&\quad \left. + \frac{(1+k)^2(n_1-i-j)(n_1-1-i-j)}{P_g^2} \right] \\
T''_{21} &= \sum_{j=0}^{c_2} \sum_{i=c_1+1-j}^{c_1+b_1-j} u_{ij}(n_1) \left[\frac{(i+j)(i+j-1)}{P_b^2} - \frac{2(1+k)(i+j)(n_1-i-j)}{P_g P_b} \right. \\
&\quad \left. + \frac{(1+k)^2(n_1-i-j)(n_1-1-i-j)}{P_g^2} \right] \\
T''_{22} &= \sum_{j=c_2+1}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_1) \left[\frac{(i+j)(i+j-1)}{P_b^2} - \frac{2(1+k)(i+j)(n_1-i-j)}{P_g P_b} \right. \\
&\quad \left. + \frac{(1+k)^2(n_1-i-j)(n_1-1-i-j)}{P_g^2} \right] \\
T''_{23} &= \sum_{j=0}^{c_2+b_2} \sum_{i=0}^{c_1+b_1-j} u_{ij}(n_2) \left[\frac{(i+j)(i+j-1)}{P_b^2} - \frac{2(1+k)(i+j)(n_2-i-j)}{P_g P_b} \right. \\
&\quad \left. + \frac{(1+k)^2(n_2-i-j)(n_2-1-i-j)}{P_g^2} \right]
\end{aligned}$$

where

$$u_{ij}(n_r) = \sum_{i=0}^{c_1-j} \frac{n_r!}{i! j! (n_r - i - j)!} k^i p_g^{n_r - i - j} p_b^{i+j}$$

The equation (7.3) is equated to zero and solved for p_b using C-program. This p_b is a portion of p^* and the other portion of p^* is $p_M = kp_b$. i.e., $p^* = kp_b + p_b$. This p^* is substituted in (6.2) to get the MAAOQ. For different DSP3($n_1, n_2, c_1, b_1, c_2, b_2$) plans, using the above procedure the MAAOQ values have been calculated and listed in Table 1.

8 Selection of DSP3 ($n_1, n_2, c_1, b_1, c_2, b_2$) for the given MAAOQ

For the given MAAOQ= 0.0075 and $k = 0.1$, from Table 1 corresponding to $k = 0.1$ search for the nearest value in MAAOQ column, one can get a three class attribute double sampling plan DSP3(30,44,2,1,1,1) which has the MAAOQ = 0.0075. The AOQ surface of this plan is presented in Figure 1.

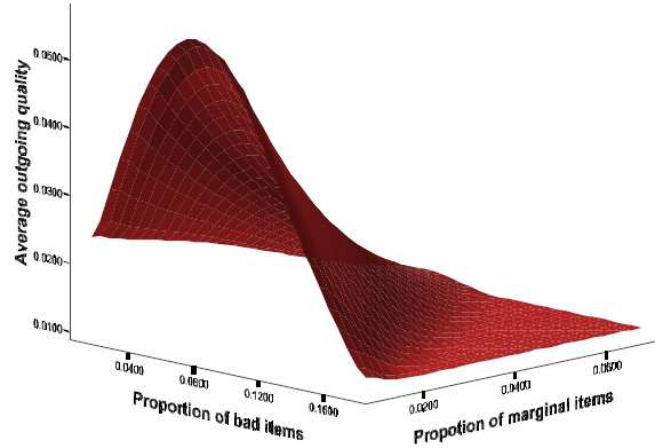


Figure 1: Average Outgoing Quality Surface of DSP3(30,44,3,2,1,1,1)

Practical Application

Suppose a fruits trading company is interested to implement the product control tool for specifying its quality standard. The company can use a suitable 3-class attributes double sampling plan stating the number of fruits to be inspected (sample size) from a lot, the maximum number of marginal and bad fruits allowed in a lot and the maximum number of bad fruits alone to be allowed.

Suppose the company fixes MAAOQ as 0.0075 (75 number of marginal and bad items out of 10000 fruits inspected) with $k = 0.1$ (10% of the bad ones are marginal) then inspect a random sample of 30 fruits from the lot of fruits and count the number of marginal and bad items (d_{11}) and number of bad items alone (d_{12}).

If $d_{11} \leq 2$ and $d_{12} \leq 1$ accept the lot of fruits; If $d_{11} > 2 + 1$ or $d_{12} > 1 + 1$ reject the lot and inform the management for corrective action.

If either (a) $2 < d_{11} \leq 2 + 1$ and $d_{12} \leq 1$ (or) (b) $d_{11} \leq 2 + 1$ and $1 < d_{12} \leq 1 + 1$ inspect another random sample of 44 fruits from the lot of fruits.

Count the total number of marginal and bad ones (d_{21}) and the number of bad fruits alone (d_{22}) in the second sample.

Let $D_1 = d_{11} + d_{21}$, the total number of marginal and bad items in the combined sample and $D_2 = d_{12} + d_{22}$, the total number of bad fruits in the combined sample.

If $D_1 \leq 2 + 1$ and $D_2 \leq 1 + 1$ then accept the lot; otherwise reject the lot and inform the management for further action.

9 Conclusion

In this paper, a procedure for constructing 3-class attributes double sampling plan indexed thorough MAAOQ is presented with suitable example. A ready-made table is also presented for easy selection of the plans. These plans are useful in the industries such as food, pharmaceutical, health insurance claim processing, etc. These plans can also be compared with the plans constructed through other parameters for their efficiency.

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References

- [1] Bray, D. F., Lyon, D. A., and Burr, I. W. (1973). Three-class Attributes Plans in Acceptance Sampling, *Technometrics*, 15(3), 575–585.
- [2] Clements, J. A. (1979). Three-class Attributes Sampling Plans, Proceedings of the ASQC Technical Conference, Atlanta.
- [3] Radhakrishnan, R. (2009). Construction of Six Sigma based Sampling Plans, Post Doctoral Thesis (DSc), submitted to Bharathiar University, Tamilnadu, India.
- [4] Radhakrishnan, R., and Ravi Sankar, S. (2008a). Selection of Three-class Attribute Single Sampling Plan for the Given Average Outgoing Quality Level, *International Journal of Statistics and Systems*, 3(2), 107–111.
- [5] Radhakrishnan, R., and Ravi Sankar, S. (2008b). Construction of Three-class Attributes Link Sampling Plan Indexed Through Acceptable Quality Level, International Symposium on Optimization and Statistics, 29th–31st Dec, Aligarh Muslim University, Aligarh, India.
- [6] Radhakrishnan, R., and Ravi Sankar, S. (2009a). Selection of Three-class Attribute Single Sampling Plan For the Given Acceptable Quality Level, *International Journal of Statistics and Management System*, 4(1–2), 150–164.
- [7] Radhakrishnan, R., and Ravi Sankar, S. (2009b). Single Sampling Plan for Three Attribute classes Indexed Through Indifference Quality Level, *College Sadhana Journal for Bloomers of Research*, 1(2), 120–127.

- [8] Radhakrishnan, R., and Ravi Sankar, S. (2009c). Three-class Attributes Single Sampling Plan Indexed Through MAPD, *National Journal of Technology*, 5(4), 70–76.
- [9] Radhakrishnan, R., and Ravi Sankar, S. (2009d). Selection of Three-class Attributes Double Sampling Plan Indexed Through Fixed Points On Oc Surface, *International Journal of Mathematics & Computations*, 2(M09), 32–42.
- [10] Soundararajan, V. (1975), Maximum Allowable Percent Defective (MAPD) Single Sampling Inspection by Attributes Plan, *Journal of Quality Technology*, 7(4), 173–182.
- [11] Suresh, K. K., and Ramkumar, T. B. (1996). Selection of a Sampling Plan indexed with Maximum Allowable Average Outgoing Quality, *Journal of Applied Statistics*, 23(6), 645–654.
- [12] Suresh, K. K., and Ravi Sankar, S. (1990). Double Sampling Plan for Three Attribute Classes, First Conference of Biometric Society, 10–11th May, University of Agricultural Sciences, Bangalore.
- [13] Suresh, K. K., Ravi Sankar, S. and Srivenkatramana. T. (1990). Link Sampling Plan for Three Attribute Classes, National Conference on Quality and Reliability, 3–6th December, Indian Institute of Technology, Bombay.

Table 1: MAAOQ values of DSP3 ($n_1, n_2, c_1, b_1, c_2, b_2$)

k	n_1	n_2	c_1	b_1	c_2	b_2	p_M	p_b	p^*	$\text{Pa}(p^*)$	MAAOQ
0.1	38	58	2	1	1	1	0.0121	0.1207	0.1328	0.0449	0.0060
	30	44	2	1	1	1	0.0151	0.1511	0.1662	0.0451	0.0075
	22	34	2	1	1	1	0.0168	0.1676	0.1843	0.0992	0.0183
	25	42	2	1	1	1	0.0133	0.1328	0.1460	0.1436	0.0210
	25	48	1	1	1	1	0.0097	0.0967	0.1063	0.2655	0.0282
	27	44	1	1	1	1	0.0056	0.0557	0.0613	0.6336	0.0388
	23	42	1	1	1	1	0.0081	0.0808	0.0889	0.4569	0.0406
	24	38	2	1	1	1	0.0078	0.0783	0.0861	0.5402	0.0465
	21	32	1	1	1	1	0.0074	0.0739	0.0813	0.6236	0.0507
	20	32	1	1	1	1	0.0084	0.0835	0.0919	0.5607	0.0515
	20	32	2	1	1	1	0.0106	0.1058	0.1164	0.4440	0.0517
0.2	32	56	2	1	1	1	0.0207	0.1036	0.1243	0.1310	0.0163
	20	30	2	1	1	1	0.0361	0.1805	0.2165	0.0912	0.0198
	20	34	1	1	1	1	0.0276	0.1379	0.1655	0.1460	0.0242
	29	60	1	1	1	1	0.0147	0.0736	0.0883	0.2847	0.0251
	21	38	1	1	1	1	0.0234	0.1172	0.1407	0.2030	0.0286
	29	54	2	1	1	1	0.0164	0.0819	0.0982	0.3181	0.0312
	28	54	1	1	1	1	0.0112	0.0562	0.0674	0.5081	0.0343
	33	48	2	1	1	1	0.0094	0.0468	0.0562	0.6798	0.0382
	26	44	2	1	1	1	0.0153	0.0767	0.0921	0.4624	0.0426
	29	42	2	1	1	1	0.0110	0.0549	0.0659	0.6600	0.0435
	24	38	2	1	1	1	0.0157	0.0784	0.0941	0.5196	0.0489
23	30	2	1	1	1	0.0133	0.0663	0.0796	0.7054	0.0562	
0.3	29	50	2	1	1	1	0.0325	0.1083	0.1407	0.1338	0.0188
	22	36	2	2	1	1	0.0470	0.1566	0.2036	0.0951	0.0194
	47	54	2	2	1	1	0.0069	0.0232	0.0301	0.8754	0.0263
	20	32	2	1	1	1	0.0457	0.1522	0.1978	0.1431	0.0283
	29	48	2	2	1	1	0.0220	0.0734	0.0955	0.4104	0.0392
	24	42	2	1	1	1	0.0270	0.0899	0.1169	0.3650	0.0427
	23	38	1	1	1	1	0.0169	0.0562	0.0731	0.6196	0.0453
	20	32	2	2	1	1	0.0360	0.1198	0.1558	0.3159	0.0492
	24	36	2	2	1	1	0.0247	0.0823	0.1070	0.4905	0.0525
	25	34	2	2	1	1	0.0205	0.0682	0.0886	0.6266	0.0555
20	30	2	1	1	1	0.0275	0.0915	0.1190	0.5233	0.0623	
0.4	30	56	1	1	1	1	0.0339	0.0848	0.1187	0.1193	0.0142
	24	42	1	1	1	1	0.0434	0.1086	0.1520	0.1073	0.0163
	40	38	2	2	1	1	0.0072	0.0179	0.0250	0.9608	0.0241
	25	44	2	1	1	1	0.0424	0.1061	0.1485	0.1921	0.0285
	25	44	2	2	1	1	0.0397	0.0993	0.1390	0.2518	0.0350
	20	38	1	2	1	1	0.0380	0.0951	0.1332	0.3103	0.0413
	23	38	2	2	1	1	0.0394	0.0984	0.1378	0.3313	0.0456
	23	34	1	1	1	1	0.0185	0.0463	0.0648	0.7166	0.0465
	22	32	1	1	1	1	0.0193	0.0484	0.0677	0.7197	0.0487
	27	36	2	1	1	1	0.0206	0.0514	0.0720	0.7195	0.0518
22	34	2	1	1	1	0.0315	0.0787	0.1102	0.5302	0.0584	