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Comparison of CRGS Plans Using Poisson and Weighted Poisson Distributions

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Abstract. Sherman (1965) has introduced a new acceptance sampling plan called Repetitive Group Sampling (RGS) plan. Ramasamy (1983) and Kuralmani (1992) further studied the RGS plans. Radhakrishnan (2004) constructed sampling plans of the type CSP-T using Maximum Allowable Percent Defective (MAPD) and Maximum Allowable Average Outgoing Quality (MAAOQ) using Poisson distribution. Radhakrishna Rao (1977) suggested a weighted binomial distribution in the construction of the sampling plans. Sudeswari (2002) constructed single sampling plan using Weighted Poisson Distribution (WPD). In this paper, the Conditional Repetitive Group Sampling (CRGS) plan is constructed with WPD as the basic distribution indexed through MAPD & MAAOQ. These plans are compared with the CRGS plans having Poisson distribution as the basic distribution suggested by Radhakrishnan (2002). Tables are provided for the easy selection of the plans.
AMS (2000) Subject Classification. 62P30.
Key Words. Operating characteristic (OC) curve, Maximum allowable percent defective, Maximum allowable average outgoing quality, Average outgoing quality.

1 Introduction

The proportion defective corresponding to the inflection point of the OC curve is interpreted as Maximum Allowable Percent Defective (MAPD). Suresh and Ramkumar (1996) have studied the construction of sampling plans through MAAOQ. The desirability of developing a set of sampling plans indexed with p^* (MAPD) has been explained by Soundararajan (1975). Mandelson (1962) has explained the desirability for developing a system of sampling plan indexed through MAPD. Mayer (1967) has suggested that MAPD (p^*) can be used as the quality standard along with some other conditions to specify OC curves.

Sherman (1965) has proposed a new type of sampling plan namely Repetitive Group Sampling (RGS) plan. The operation of the plan is similar to that of the sequential sampling plan. According to Sherman, the RGS plan gives minimum sample size as well as desired protection. It is usually more efficient between the single and sequential sampling plans, *i.e.*, these plans give an intermediate sample size between the single sampling plans and sequential sampling plans.

Ramasamy (1983) and Kuralmani (1992) have made contributions in the construction of RGS plans. Govindaraju (1987) has shown that the OC functions of the RGS plan of Sherman (1965), a single sampling quick switching system (QSS-1) of Romboski (1969) and the dependant stage sampling plan of Wortham & Mogg (1970) are essentially the same. Selection of RGS plan involving the minimum sum of risks was carried out by Subramani (1991). Radhakrishnan (2004) studied Construction of the sampling plan of the type CSP-T using MAPD and MAAOQ. Radhakrishnan and Sampathkumar (2005, 2006 and 2007) studied mixed sampling plans through MAPD, AQL, IQL and MAAOQ with RGS as the basic plan. The weighted binomial distribution was studied by Radhakrishna Rao (1977) and outlined its uses in the construction of sampling plans. Sudeswari (2002) studied the designing of sampling plan using weighted Poisson distribution as the basic distribution. Radhakrishnan and Mohana Priya (2008) constructed the single and double sampling plans using conditional weighted Poisson distribution.

2 Glossary of symbols

p	—	Quality of submitted lots
p^*	—	Maximum Allowable Perceent Defective
Pa (p)	—	Probability of acceptance of the lot quality p
n	—	Sample size
d	—	Number of defectives counted
c	—	Acceptance number
R	—	Ratio of MAAOQ to MAPD

3 Definition of MAAOQ

The MAAOQ (Maximum Allowable Average Outgoing Quality) of the sampling plan is defined by the Average Outgoing Quality (AOQ) at MAPD.

$$\text{i.e.,}\quad \text{AOQ} = p\ \text{Pa}\ (p).$$

Thus MAAOQ = AOQ at $p = p^*$. This can be written as MAAOQ = p^* Pa (p^*).

3.1 Conditions for the application of CRGS with WPD in the product control

- Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots submitted may be isolated or series.
- Inspection is by attributes, with the lot quality defined as the proportion defective.
- Variation in the lot quality may exist.
- Lot has at least one defective unit.
- Lots submitted for inspection may be of seconds quality.

3.2 Operating procedure.

Step.1 From each of the submitted lots, select a sample of size n and observe the number of non-conformities (say d)

Step.2 Accept the current lot if $d \leq c_1$, reject the lot, if $d > c_2$

Step.3 If $c_1 < d \leq c_2$, utilize the information of the next proceeding lot (*i.e.*) the current lot is accepted if the proceeding lot result shows $d \leq c_1$ in the sample, in case the proceeding lot result also shows $c_1 < d \leq c_2$, then utilize next proceeding lot and checkup whether $d \leq c_1$ or $d > c_2$ continue utilizing the proceeding lot results till satisfying $d \leq c_1$ or $d > c_2$.

3.3 Operating characteristic function

The probability mass function of weighted Poisson distribution is given by,

$$P(X:\lambda,\alpha) = \frac{X^\alpha P(X,\lambda)}{\sum X^\alpha P(X,\lambda)},\ X = 0, 1, \ldots \quad \text{where } \lambda = np$$

The probability mass function of weighted Poisson distribution for $\alpha = 1$ is given by

$$\begin{aligned} P(x:\lambda) &= P(X:\lambda,\alpha), \quad \alpha = 1 \\ &= \frac{e^{-np}(np)^{x-1}}{(x-1)!}, \quad x = 1, 2, \ldots \end{aligned}$$

OC function of conditional RGS using weighted Poisson distribution is as follows:

$$Pa(p) = P_1/(1 - P_1P_3)$$

where

$$P_1 = \sum_{x=1}^{c_1} \frac{e^{-np}(np)^{x-1}}{(x-1)!}$$
$$P_2 = 1 - \sum_{x=1}^{c_2} \frac{e^{-np}(np)^{x-1}}{(x-1)!}$$
$$P_3 = 1 - P_1 - P_2$$

The AOQ function for conditional repetitive group sampling plan is given by

$$\text{AOQ } (p) = p \cdot \text{Pa } (p)$$
$$\text{MAAOQ} = \text{AOQ } (p^*)$$

3.4 Construction of the plans

The value of MAPD (p^*) is obtained using $d^2 \text{ Pa}(p)/dp^2 = 0$, at $p = p^*$ and $d^3\text{Pa } (p)/dp^3 \neq 0$, at $p = p^*$. The values of n MAPD $= np^*$ and n MAAOQ $= np^*$ Pa (p^*), where Pa (p) is the probability of acceptance at $p = p^*$ and $R =$ MAPD/MAAOQ have been calculated for different possible combinations of c_1 and c_2 using a C++ program and presented in Table 3.

For a specified MAPD and MAAOQ = AOQL, Table 4 is used to construct the plan. $R =$ MAPD/MAAOQ and $R_1 =$ MAPD/AOQL are found out. The value nearer to the calculated value is obtained. The corresponding values of c_1 and c_2 are noted. From this one can find the parameters of CRGS.

Selection of sampling plan through MAPD and MAAOQ

Table 3 is used to construct the plan when the MAPD and MAAOQ are specified. One can find the ratio $R =$ MAPD/MAAOQ which is the function of c_1 and c_2 and the values are obtained from the column R in Table 3. The corresponding values of c_1 and c_2 are noted, and hence the parameter of n, c_1 and c_2 for the repetitive group sampling plan is determined.

Example.1: Given MAPD = 0.0196 & MAAOQ = 0.0142 the ratio $R =$ MAPD/MAAOQ = 1.381 and the corresponding value of n, c_1 & c_2 of the sampling plans are respectively $n = 31$, $c_1 = 1$ & $c_2 = 4$. Then the CRGS with weighted Poisson distribution is $n = 31$, $c_1 = 1$ & $c_2 = 4$ with specified MAPD = 0.0196 and MAAOQ = 0.0142. The OC curve for the plan is presented in figure 1. More examples are provided in Table 1.

Table 1: CRGS plans for a specified MAPD and MAAOQ

MAPD	MAAOQ	n	c_1	c_2
0.03	0.0207	47	2	3
0.026	0.019	22	1	3
0.044	0.0296	36	2	4
0.022	0.0149	91	3	3
0.040	0.026	67	3	6
0.065	0.042	51	4	5
0.015	0.0096	234	4	6
0.025	0.0154	200	6	6
0.081	0.051	57	5	8
0.037	0.0234	116	5	6

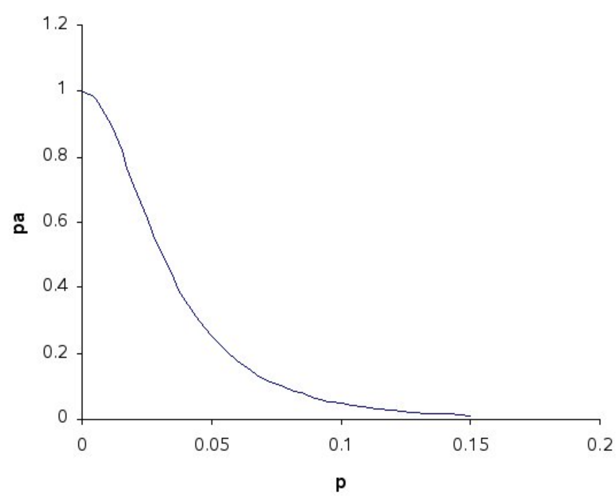

Figure 1: OC curve of the CRGS using WPD for $n = 31$, $c_1 = 1$ & $c_2 = 4$.

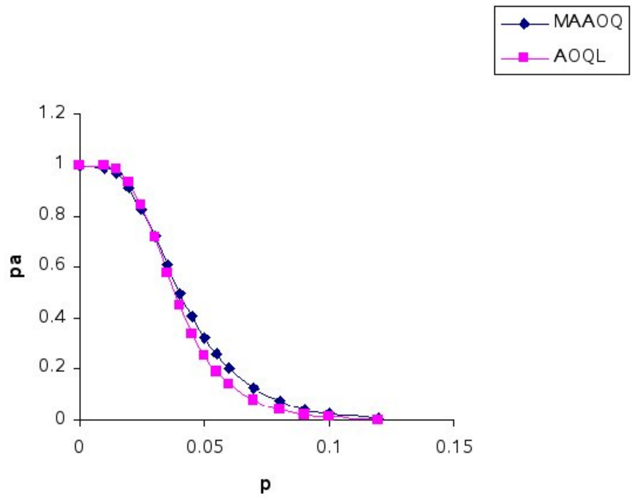

Figure 2: OC curves for the plans indexed through MAAOQ and AOQL

Selection of the plan through MAPD and AOQL

For a specified MAPD and MAAOQ = AOQL, Table 4 is used to construct the plan. R = MAPD/MAAOQ and R_1 = MAPD/AOQL are found out. The value nearer to the calculated value is obtained. The corresponding values c_1 and c_2 are noted, and hence the parameters n, c_1 and c_2 for the CRGS is determined.

Example.2: Given MAPD = 0.0326 and MAAOQ = AOQL = 0.0216 compute the ratio R = MAPD/MAAOQ = 1.509 and R_1 = MAPD/AOQL = 1.509. From Table 4, the nearest value of 1.509 is $R = 1.509$ with $c_1 = 3$ and $c_2 = 4$, nMAAOQ = 1.5547 and $c_1 = 3$, $c_2 = 6$ nAOQL = 1.7651 respectively. The value of n is associated with them are respectively n = n MAAOQ/MAAOQ = 1.5547/0.0216 = 72, and n = nAOQL/AOQL = 1.7651/0.0216 = 82. Hence the repetitive group sampling plans for specified MAPD = 0.0326 and MAAOQ = 0.0216 and $n = 72$, $c_1 = 3$, $c_2 = 4$ and the CRGS plan for specify MAPD = 0.036 and AOQL = 0.0216 is $n = 82$, $c_1 = 3$, $c_2 = 6$. The OC curves are presented in figure 2.

Practical Problem

These plans can be applied in industries such as manufacturing, service, catering and so on.

Problem.1: A Textile company producing cotton and polyester yarn fixes the MAPD (p^*) as 0.09 and MAAOQ as 0.058 then the corresponding sampling plan is $n = 26$, $c_1 = 3$ and $c_2 = 6$.

Problem.2: A Health drink manufacturing company can fix MAPD and

MAAOQ values respectively 0.0196 and 0.0142 then the corresponding sampling plan is $n = 31$, $c_1 = 1$ and $c_2 = 4$.

4 Comparison of CRGS plan indexed through Poisson and Weighted Poisson distribution

In this section the CRGS plans indexed through Poisson distribution (Radhakrishnan, 2002) is compared with CRGS plan indexed through weighted Poisson distribution. For the different values of (MAPD, MAAOQ) and (MAPD, AOQL) the values of n, c_1, c_2 (indexed through Poisson) and n, c_1, c_2 (indexed through WPD) are presented in Table 2.

Construction of OC curve

In constructing OC curves for the plans 'np' and Pa (p) values are calculated for different values of 'p' for the plans $n = 30$, $c_1 = 2$, $c_2 = 5$ (indexed through Poisson) and $n = 26$, $c_1 = 3$, $c_2 = 6$ (indexed through WPD) and presented in figure 3.

5 Conclusion

In this paper a procedure for the selection of CRGS plans with weighted Poisson distribution as a basic distribution indexed through MAPD and MAAOQ is presented. These types of plans are very much essential for the floor engineers to accept or reject the lots having at least one defective in the lot. It is found that the size of the sample is less in the construction of the sampling plans using weighted Poisson distribution than Poisson distribution. Further it is derived that the constructions of sampling plans indexed through MAAOQ have lesser sample size than indexed through AOQL, which was established by Radhakrishnan (2002). It is concluded from the study that the size of the sample is less in the construction of the sampling plans indexed through MAAOQ than indexed through AOQL irrespective of the basic distribution whether it is Poisson or Weighted Poisson Distribution.

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Table 2: Comparison of CRGS plan

Given values			Poisson distribution (Radhakrishnan, 2002)			Weighted Poisson Distribution		
MAPD	MAAOQ	AOQL	n	c_1	c_2	n	c_1	c_2
0.09	0.058	–	30	2	5	26	3	6
0.09	–	0.058	34	3	3	32	4	4
0.0326	0.022	–	48	1	3	43	2	3
0.0326	–	0.022	48	1	3	44	2	3

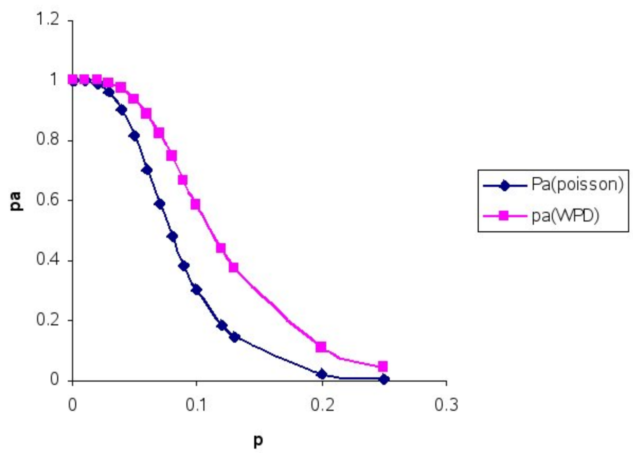

Figure 3: OC curves for CRGS

Table 3: Certain parametric (nMAPD, nMAAOQ) values for CRGS plan

c_1	c_2	nMAPD	nMAAOQ	R
1	2	0.4822	0.3636	1.326
1	3	0.5842	0.4245	1.376
1	4	0.6030	0.4366	1.381
2	2	1.000	0.7358	1.359
2	3	1.3971	0.9668	1.445
2	4	1.5712	1.0587	1.484
2	5	1.6548	1.0967	1.508
3	3	2.0000	1.3534	1.478
3	4	2.3468	1.5547	1.509
3	5	2.5448	1.6645	1.529
3	6	2.6656	1.7239	1.546
3	7	2.7276	1.7498	1.559
4	4	3.0000	1.9416	1.545
4	5	3.3115	2.1402	1.547
4	6	3.5190	2.2509	1.563
4	7	3.6616	2.3336	1.569
4	8	3.7490	2.3647	1.585
5	5	4.0000	2.5152	1.590
5	6	4.2848	2.7095	1.581
5	7	4.4956	2.8447	1.580
5	8	4.6518	2.9313	1.587
5	9	4.7580	2.9677	1.603
6	6	5.0000	3.08	1.623
6	7	5.2636	3.2915	1.599
6	8	5.4746	3.4261	1.597
6	9	5.6396	3.5212	1.602
6	10	5.7598	3.5707	1.613
7	7	6.0000	3.6378	1.649
7	8	6.2461	3.8495	1.623
7	9	6.4558	4.0026	1.612
7	10	6.6269	4.1023	1.615

Table 4: Parameters nMAAOQ, nAOQL of CRGS using WPD ($\alpha = 1$)

c_1	c_2	nMAAOQ	nAOQL	R =MAPD/MAAOQ	R_1 =MAPD/AOQL
1	2	0.3636	0.4302	1.326	1.121
1	3	0.4245	0.4668	1.376	1.251
1	4	0.4366	0.4788	1.381	1.259
2	2	0.7358	0.8400	1.359	1.190
2	3	0.9668	0.9733	1.445	1.435
2	4	1.0587	1.0610	1.484	1.481
2	5	1.0967	1.1007	1.508	1.502
3	3	1.3534	1.3711	1.478	1.459
3	4	1.5547	1.5615	1.509	1.503
3	5	1.6645	1.6943	1.529	1.504
3	6	1.7239	1.7651	1.546	1.510
3	7	1.7498	1.7957	1.559	1.519
4	4	1.9416	1.9424	1.545	1.544
4	5	2.1402	2.1794	1.547	1.519
4	6	2.2509	2.3511	1.563	1.497
4	7	2.3336	2.4524	1.569	1.493
4	8	2.3647	2.5023	1.585	1.498
5	5	2.5152	2.5435	1.590	1.573
5	6	2.7095	2.8196	1.581	1.519
5	7	2.8447	3.0253	1.580	1.486
5	8	2.9313	3.1553	1.587	1.474
5	9	2.9677	3.2250	1.603	1.475
6	6	3.08	3.1682	1.623	1.578
6	7	3.2915	3.4770	1.599	1.514
6	8	3.4261	3.7133	1.597	1.474
6	9	3.5212	3.8698	1.602	1.457
6	10	3.5707	3.9611	1.613	1.454
7	7	3.6378	3.8120	1.649	1.574
7	8	3.8495	4.1506	1.623	1.505
7	9	4.0026	4.1136	1.612	1.569
7	10	4.1023	4.5941	1.615	1.442

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