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Joint Confidence Regions for the Parameters of the Weibull Distribution Based on Records

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Abstract. In this paper, we consider joint confidence regions for the parameters of Weibull distribution based on record statistics. One of the applications of the joint confidence regions of the parameters is to find confidence bounds for the functions of the parameters. Joint confidence regions for the parameters of extreme value distribution are also discussed. Two numerical examples with real data set and simulated data, are presented to illustrate the proposed method. A simulation study is performed to compare the proposed joint confidence regions.

1 Introduction

Let $X_1,\ X_2, ...$ be a sequence of independent and identically distributed ($i.i.d$) random variables with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$. An observation X_j will be called an upper (lower) record value if its value exceeds (is lower than) that of all previous observations. Thus, X_j is an upper (lower) record if $X_j > (<) X_i$ for every $i < j$. If $\{U(m), m \geq 1\}$ is defined by

$$U(1) = 1, \qquad U(m) = \min\{j : j > U(m-1), X_j > X_{U(m-1)}\},$$

for $m \geq 2$, then the sequence $\{X_{U(m)}, m \geq 1\}$ provides a sequence of upper record statistics. The sequence $\{U(m), m \geq 1\}$ represents the record times.

Record data arise in a wide variety of practical situations. Examples include industrial stress testing, hydrology, seismology, sporting and athletic events, and oil and mining surveys. The statistical study of record values started with Chandler (1952) and has now spread in different directions. For more details and applications of record values, one may refer to Arnold et al. (1998).

The Weibull distribution is widely used in reliability analysis, because of its flexibility in modeling both increasing and decreasing failure rates. There are numerous papers and books dealing with various aspects of Weibull modeling, inference, applications, as well as parameter estimation. The cumulative distribution function (cdf) and probability density function (pdf) of a two-paramcter Weibull distribution are given, respectively, by

$$F(x;\theta,\beta) = 1 - e^{-(\frac{x}{\theta})^{\beta}}, \quad x > 0, \tag{1.1}$$

and

$$f(x;\theta,\beta) = \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1}e^{-(\frac{x}{\theta})^{\beta}}, \quad x > 0, \tag{1.2}$$

where θ is the scale parameter and β is the shape parameter. This model includes the exponential distribution with constant failure rate for $\beta = 1$ and provide an increasing failure rate for $\beta > 1$ and decreasing failure rate for $\beta < 1$. For the Weibull distribution, the reliability function is given by

$$R(t) = \exp[-(t/\theta)^{\beta}],$$

which is a function of the parameters θ and β. Hence the joint confidence regions for θ and β are important because they can be used to find confidence bounds for any function of the parameters θ and β such as the reliability function $R(t)$.

Confidence intervals for the parameters θ and β were discussed by many authors. See, for examples, Billman and Antle (1972), Chao and Hwang (1986), Cohen and Whitten (1988), Lawless (1983) and Bain and Engelhardt (1991). Chen (1998) obtained a joint confidence region for the parameters θ and β based on the first k order statistics of a sample of size n. Wu (2002) discussed an exact confidence interval and an exact joint confidence region for the parameters θ and β based on Type-II progressively censored data. Tse and Xiang (2003) discussed seven different confidence interval-estimation procedures for parameters of Weibull distribution based on Type-II progressively censored data with random removals. Wu and Tseng (2006) provided some pivotal quantities to test and establish confidence interval of the shape parameter β on the basis of the first m observed upper record values. In this paper, we extend the idea presented in Wu and Tseng (2006) to obtain exact joint confidence regions for the parameters of Weibull distribution based on the first m observed upper record values.

The paper is organized as follows. We derive $m - 1$ exact joint confidence regions for the parameters θ and β in Section 2 and also we present $m-1$ exact joint confidence regions for the parameters of the extreme value distribution. Based on the joint confidence region of the parameters, we also obtain a lower confidence bound for the reliability function. In Section 3, we present two numerical examples as well as a Monte Carlo simulation study to illustrate the proposed joint confidence regions discussed in this paper.

2 Main Results

Let $X_{U(1)} < X_{U(2)} < \cdots < X_{U(m)}$ be the first m upper record values from (1.1). For notation simplicity, we will write X_i for $X_{U(i)}$. It can be shown that if let $Y_i = (\frac{X_i}{\theta})^\beta$, $(i = 1, 2, \ldots, m)$, then $Y_1 < Y_2 < \cdots < Y_m$ are the first m upper record values from a standard exponential distribution. Moreover, the spacings

$$\begin{aligned} &S_1 = Y_1, \\ &S_2 = Y_2 - Y_1, \\ &\vdots \\ &S_m = Y_m - Y_{m-1}, \end{aligned} \tag{2.1}$$

are *i.i.d.* random variables from a standard exponential distribution. This is a known result about the first m upper records from the standard exponential distribution (see Arnold et al. (1998)). Hence

$$V_j = 2 \sum_{i=1}^{j} S_i = 2\ Y_j$$

has a chi-square distribution with $2j$ degrees of freedom and

$$U_j = 2 \sum_{i=j+1}^{m} S_i = 2\ (Y_m - Y_j)$$

has a chi-square distribution with $2(m-j)$ degrees of freedom. We can also see that U_j and V_j are independent random variables. Let

$$T_j = \frac{U_j/2(m-j)}{V_j/2j} = \frac{j\ U_j}{(m-j)V_j} = \frac{j}{m-j}\left(\frac{Y_m - Y_j}{Y_j}\right), \tag{2.2}$$

and

$$S = U_j + V_j = 2Y_m. \tag{2.3}$$

It is easy to show that T_j has an F distribution with $2(m-j)$ and $2j$ degrees of freedom and S has a chi-square distribution with $2m$ degrees of freedom. Furthermore, T_j and S are independent for each j (Johnson et al. (1994), Page 350).

Let $F_\alpha(\upsilon_1, \upsilon_2)$ denote the upper α percentile of F distribution with υ_1 and υ_2 degrees of freedom. Let $\chi^2_\alpha(\upsilon)$ be the percentile of chi-square distribution with right-tail probability α and υ degrees of freedom. Exact joint confidence regions for the parameters β and θ for $j = 1, 2, \ldots, m-1$ are given in the following theorem.

Theorem 1. Suppose that $X_1 < X_2 < \cdots < X_m$ be the first m observed upper record values from Weibull distribution. Then, the following inequalities determine $100(1-\alpha)\%$ joint confidence regions for θ and β, for $j = 1, 2, \ldots, m-1$:

$$\begin{cases} \dfrac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \dfrac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}, \\ X_m\left(\dfrac{2}{\chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2m)}\right)^{\frac{1}{\beta}} < \theta < X_m\left(\dfrac{2}{\chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m)}\right)^{\frac{1}{\beta}}. \end{cases}$$

where $0 < \alpha < 1$.

Proof. From (2.2) and (2.3), we know that

$$T_j(\beta) = \frac{j}{m-j}\left(\frac{Y_m - Y_j}{Y_j}\right) = \frac{j}{m-j}\left[\frac{X_m^\beta}{X_j^\beta} - 1\right], \quad j = 1, 2, \ldots, m-1,$$

has an F distribution with $2(m-j)$ and $2j$ degrees of freedom, and

$$S = 2Y_m = 2\left(\frac{X_m}{\theta}\right)^\beta$$

has a chi-square distribution with $2m$ degrees of freedom, and it is independent of $T_j(\beta)$, $(j = 1, 2, \ldots, m-1)$. Next, for $0 < \alpha < 1$, we have

$$P\left[F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j), 2j) < T_j(\beta) < F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j), 2j)\right] = \sqrt{1-\alpha},$$

and

$$P\left[\chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m) < S < \chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2m)\right] = \sqrt{1-\alpha}.$$

From these relationships, we obtain

$$P\Bigg(F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j), 2j) < T_j(\beta) < F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j), 2j)\ , \quad \chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m) < 2\left(\frac{X_m}{\theta}\right)^\beta < \chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2m)\Bigg) = 1-\alpha$$

Equivalently,

$$P\left\{\frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j), 2j) + 1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j), 2j) + 1\right]}{\ln\left(\frac{X_m}{X_j}\right)}, \quad X_m\left(\frac{2}{\chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2m)}\right)^{\frac{1}{\beta}} < \theta < X_m\left(\frac{2}{\chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m)}\right)^{\frac{1}{\beta}}\right\} = 1-\alpha$$

Then the theorem follows. □

As an application, Theorem 1 will be used in the next theorem to obtain a lower confidence bound for the reliability function of a two-parameter Weibull distribution. Note that the reliability function $R(t) = \exp[-(t/\theta)^{\beta}]$ is increasing in θ. Thus, the next corollary is useful for that purpose.

Corollary 1. Suppose that $X_1 < X_2 < \cdots < X_m$ be the first m observed upper record values from Weibull distribution. Then, the following inequalities determine $100(1-\alpha)\%$ joint confidence regions for θ and β for $j = 1, 2, \ldots, m-1$:

$$\begin{cases} \dfrac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \dfrac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}, \\ \theta > X_m\left(\dfrac{2}{\chi^2_{\sqrt{1-\alpha}}(2m)}\right)^{\frac{1}{\beta}}. \end{cases}$$

where $0 < \alpha < 1$.

Theorem 2. Suppose that $X_1 < X_2 < \cdots < X_m$ be the first m upper record values from the Weibull distribution with cdf (1.1). Then for any $0 < \alpha < 1$,

$$\inf_{\beta} \exp\left[-\frac{t^{\beta}\chi^2_{\sqrt{1-\alpha}}(2m)}{2X_m^{\beta}}\right],$$

is a $(1-\alpha)100\%$ lower confidence bound for the reliability function $R(t)$, where the infimum is taken over the region

$$\left(\frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}, \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}\right)$$

Proof. For fixed t and $\beta > 1$, $R(t) = \exp[-(t/\theta)^{\beta})]$ is increasing in θ, then

$$P\left[R(t) > \inf_{\beta}\left\{\exp\left[-\frac{t^{\beta}\chi^2_{\sqrt{1-\alpha}}(2m)}{2X_m^{\beta}}\right] : \right.\right.$$
$$\left.\left.\frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}\right\}\right]$$

$$
\begin{aligned}
&= P\left[R(t) > \inf_{\beta}\left\{\exp\left[-(\frac{t}{\theta})^{\beta}\right] : \theta = X_m\left(\frac{2}{\chi^2_{\sqrt{1-\alpha}}(2m)}\right)^{\frac{1}{\beta}},\right.\right. \\
&\qquad \left.\left.\frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}\right\}\right] \\
&= P\left[R(t) > \inf_{\beta}\left\{\exp\left[-(\frac{t}{\theta})^{\beta}\right] : \theta > X_m\left(\frac{2}{\chi^2_{\sqrt{1-\alpha}}(2m)}\right)^{\frac{1}{\beta}},\right.\right. \\
&\qquad \left.\left.\frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}\right\}\right] \\
&= P\left[\theta > X_m\left(\frac{2}{\chi^2_{\sqrt{1-\alpha}}(2m)}\right)^{\frac{1}{\beta}}, \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)} < \beta < \right. \\
&\qquad \left. \frac{\ln\left[\left(\frac{m-j}{j}\right)F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2j)+1\right]}{\ln\left(\frac{X_m}{X_j}\right)}\right] \\
&= 1-\alpha.
\end{aligned}
$$

The proof is thus completed. □

The result in Theorem 1 can be used for constructing some exact joint confidence regions for the parameters of extreme value distribution. It is known that, if the random variable X has a Weibull distribution in (1.1), then $Z = \ln X$ has the extreme value distribution with cdf as

$$
G(z) = 1 - \exp\left[-\exp(\frac{z-\mu}{\sigma})\right], \quad -\infty < z < \infty, \tag{2.4}
$$

where $\mu = \ln(\theta)$ and $\sigma = \frac{1}{\beta}$. The above distribution is known as an extreme value distribution with the location and scale parameters μ and σ, respectively.

In the next theorem, $m-1$ exact joint confidence regions for the parameters of extreme value distribution are given. The proof is easy and omitted

Theorem 3. Suppose that $Z_1 < Z_2 < \cdots < Z_m$ be the first m observed upper record values from extreme value distribution in (2.4). Then, the following inequalities determine $100(1-\alpha)\%$ joint confidence regions for μ and σ, with

$j = 1, 2, \ldots, m-1$:

$$\begin{cases} \dfrac{Z_m - Z_j}{\ln\left[\left(\frac{m-j}{j}\right) F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j), 2j) + 1\right]} < \sigma < \dfrac{Z_m - Z_j}{\ln\left[\left(\frac{m-j}{j}\right) F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j), 2j) + 1\right]}, \\ Z_m - \sigma \ln\left(\dfrac{\chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2m)}{2}\right) < \mu < Z_m - \sigma \ln\left(\dfrac{\chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m)}{2}\right). \end{cases}$$

where $0 < \alpha < 1$.

3 Numerical Computations

In this section, two examples are given to illustrate the proposed joint confidence regions. We apply the proposed methods to one of practical data set and another simulated data set. Further, a Monte Carlo simulation is conducted to compare the proposed joint confidence regions in the sense of coverage probability and confidence area for different sample sizes and for different parameter values.

3.1 Example 1. (Real data set)

The following upper record values for the month of October from the data that Roberts (1979) has given the monthly and annual maximal of one-hour mean concentration of sulfur dioxide (in pphm) from Long Beach, California, for the years 1956 to 1974:

26 27 40 41.

Chan (1993) indicated that the Weibull destruction is a reasonable good model for this data set (also see Chan (1998)). Here $m = 4$, so we obtain three joint confidence regions of the parameters β and θ based on the three couple pivotal quantities $(T_1, S), (T_2, S)$ and (T_3, S). To obtain 95% joint confidence regions for β and θ, we need the following percentiles:

$$F_{0.0127(6,2)} = 78.07254\ ,\ F_{0.9873(6,2)} = 0.10144,$$

$$F_{0.0127(4,4)} = 14.02461\ ,\ F_{0.9873(4,4)} = 0.07130,$$

$$F_{0.0127(2,6)} = 9.85839\ ,\ F_{0.9873(2,6)} = 0.01281,$$

$$\chi^2_{0.0127(8)} = 19.43470\ ,\ \chi^2_{0.9873(8)} = 1.76871.$$

By Theorem 1, the 95% joint confidence regions for θ and β are:

$$A_1 = \left\{(\theta, \beta) : 0.583 < \beta < 11.989, 41(0.103)^{1/\beta} < \theta < 41(0.131)^{1/\beta}\right\}$$

$$A_2 = \left\{(\theta, \beta) : 0.165 < \beta < 6.487, 41(0.103)^{1/\beta} < \theta < 41(0.131)^{1/\beta}\right\}$$

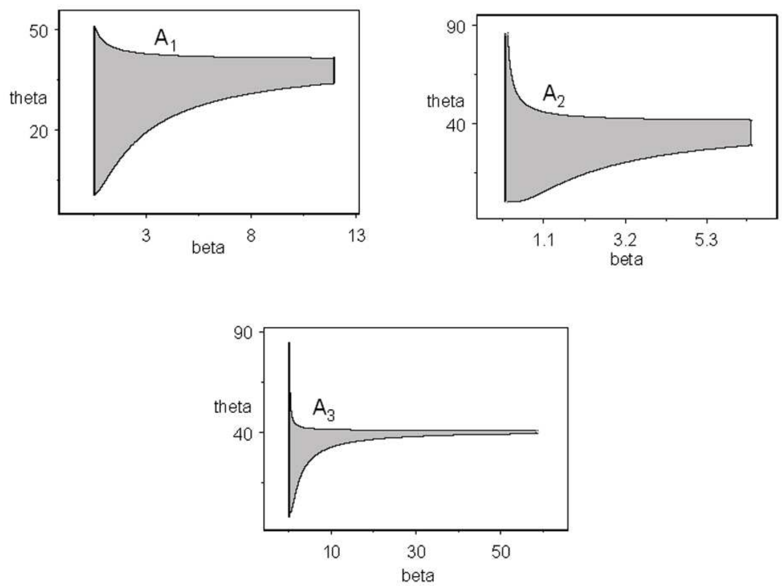

Figure 1: Joint confidence regions for θ and β in Example 1.

$$A_2 = \left\{(\theta,\beta) : 0.173 < \beta < 58.940, 41(0.103)^{1/\beta} < \theta < 41(0.131)^{1/\beta}\right\}$$

Figure 1 shows the above joint confidence regions for θ and β. The area of above joint confidence regions are 194.906, 166.421, 369.024, respectively. Thus in this example, A_2 is the optimal joint confidence region for the parameters θ and β.

3.2 Example 2. (Simulated data)

In this example, we consider a simulated record values of size $m = 5$ from the Weibull distribution in (1.1) with parameters $\beta = 1$ and $\theta = 1$. The simulated observations are as follows:

`0.0612    0.7153    1.0332    1.2179    1.5962.`

To find 95% joint confidence regions for β and θ, we need the following percentiles:

$$F_{0.0127(8,2)} = 78.11416 \ , \ F_{0.9873(8,2)} = 0.12634,$$
$$F_{0.0127(6,4)} = 13.36579 \ , \ F_{0.9873(6,4)} = 0.12049,$$
$$F_{0.0127(4,6)} = 8.29918 \ , \ F_{0.9873(4,6)} = 0.07482,$$
$$F_{0.0127(2,8)} = 7.91541 \ , \ F_{0.9873(2,8)} = 0.01281,$$
$$\chi^2_{0.0127(10)} = 22.51156 \ , \ \chi^2_{0.9873(10)} = 2.71822.$$

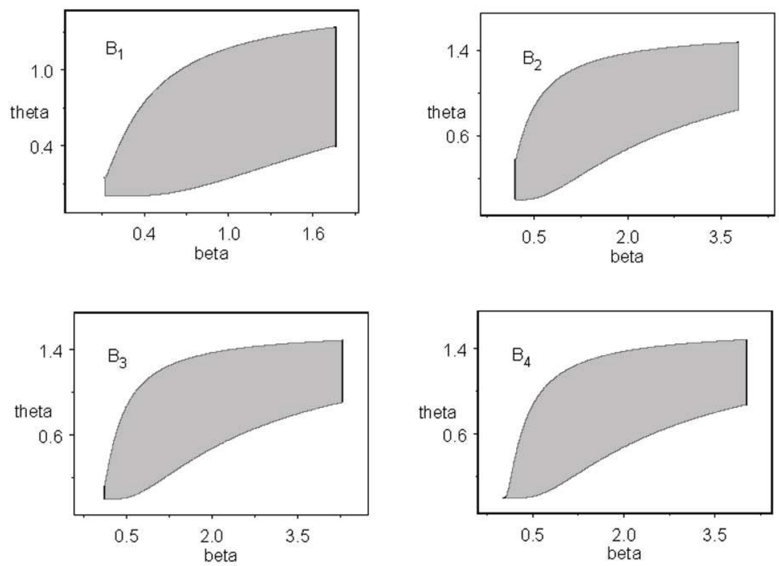

Figure 2: Joint confidence regions for θ and β in Example 2.

By Theorem 1, the 95% joint confidence regions for θ and β are:

$$B_1 = \left\{(\theta, \beta) : 0.125 < \beta < 1.763, 1.5962(0.089)^{1/\beta} < \theta < 1.5962(0.736)^{1/\beta}\right\}$$

$$B_2 = \left\{(\theta, \beta) : 0.207 < \beta < 3.796, 1.5962(0.089)^{1/\beta} < \theta < 1.5962(0.736)^{1/\beta}\right\}$$

$$B_3 = \left\{(\theta, \beta) : 0.112 < \beta < 4.314, 1.5962(0.089)^{1/\beta} < \theta < 1.5962(0.736)^{1/\beta}\right\}$$

$$B_4 = \left\{(\theta, \beta) : 0.012 < \beta < 4.035, 2.4472(0.089)^{1/\beta} < \theta < 1.5962(0.736)^{1/\beta}\right\}$$

The area of above joint confidence regions are 1.4596, 3.0028, 3.3372, 3.1750, respectively. Thus in this example B_1 and B_2 are the optimal joint confidence regions for parameters θ and β. Figure 2 shows the above joint confidence regions.

3.3 Simulation

In this section, we carry out a Monte Carlo simulation to compare the proposed joint confidence regions discussed in this paper. In this simulation, we randomly generated upper record sample $X_1, X_2, \ldots, X_m$ from the Weibull distribution with the values of parameters $(\theta, \beta) = (1, 1), (1, 1.5)$ and $(1.5, 2.5)$, and then computed 95% joint confidence regions using Theorem 1. We replicated the process 5000 times. We presented in Table 1, the simulated average confidence area. From Table 1, we observe that the average confidence area decrease when m is increasing. In addition, if we consider $m-1$ couple pivotal quantities $(T_1, S), (T_2, S), ..., (T_{m-1}, S)$ to establish confidence area for the parameters θ and β, we observe that the two couple pivotal quantities $(T_{[\frac{m}{5}]}, S)$ and $(T_{[\frac{m}{5}]+1}, S)$, provide the smallest confidence areas in most cases considered.

For purpose of validations, we also presented in Table 1, the 95% coverage probabilities of the proposed joint confidence regions. From Table 1, we observe that the coverage probabilities of the joint confidence regions for (θ, β) are close to the desired level of 0.95 for different parameters and sample sizes. Hence, our proposed methods for constructing exact confidence regions can be used reliably.

Table 1. The simulated average confidence area (CA) and 95% coverage probabilities (CP).

		(θ,β)=(1, 1)		(θ,β)=(1, 1.5)		(θ,β)=(1.5, 2.5)	
m	j	CA	CP	CA	CP	CA	CP
5	1	8.3960	0.9538	6.4587	0.9488	8.2579	0.9502
	2	8.9606	0.9504	6.7375	0.9474	8.5973	0.9540
	3	10.9226	0.9504	7.8628	0.9484	9.8892	0.9542
	4	16.6913	0.9472	11.2085	0.9500	13.5125	0.9508
7	1	6.0920	0.9506	4.5747	0.9488	5.8038	0.9538
	2	5.8477	0.9468	4.3971	0.9502	5.7006	0.9514
	3	6.3028	0.9546	4.7080	0.9548	6.0131	0.9518
	4	7.3868	0.9456	5.3572	0.9478	6.7674	0.946
	5	9.4195	0.9512	6.5515	0.9536	8.0811	0.9446
	6	15.8428	0.9500	9.9534	0.9474	11.1210	0.9538
10	1	4.6583	0.9542	3.4174	0.9550	4.2980	0.9452
	2	4.1509	0.9484	3.0698	0.9510	3.9150	0.9476
	3	4.1598	0.9498	3.1098	0.9542	3.9639	0.9504
	4	4.3357	0.9532	3.2364	0.9508	4.1447	0.9458
	5	4.8326	0.9464	3.5137	0.9472	4.3933	0.9560
	6	5.5526	0.9480	3.9421	0.9518	4.8324	0.9536
	7	7.0399	0.9478	4.6466	0.9526	5.5904	0.948
	8	9.3959	0.9454	5.8117	0.9518	6.6222	0.9488
	9	16.3633	0.9420	9.2976	0.9516	9.5545	0.9452
15	1	3.5319	0.9500	2.5577	0.9508	3.1271	0.9502
	2	2.9014	0.9520	2.1475	0.9472	2.7136	0.9460
	3	2.7433	0.9482	2.0398	0.9506	2.6045	0.9504
	4	2.7004	0.9506	2.0173	0.9490	2.6188	0.9482
	5	2.7291	0.9540	2.0763	0.9504	2.6472	0.9546
	6	2.9138	0.9540	2.1507	0.9526	2.7327	0.9514
	7	3.0995	0.9526	2.2265	0.9506	2.8533	0.9524
	8	3.4257	0.9490	2.4273	0.9538	3.0522	0.9520
	9	3.8157	0.9450	2.6308	0.9462	3.2589	0.9448
	10	4.2898	0.9554	2.9372	0.9490	3.5571	0.9492
	11	5.2670	0.9506	3.3982	0.9486	3.9739	0.9508
	12	6.5863	0.9526	4.1045	0.9498	4.5679	0.9468
	13	9.3341	0.9468	5.3524	0.9452	5.6203	0.9476
	14	17.448	0.9520	8.8399	0.9482	8.3989	0.9438

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